Logarithms - Modelling Exponential Data

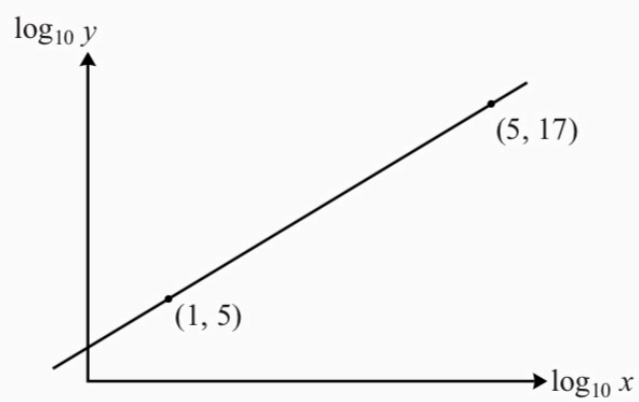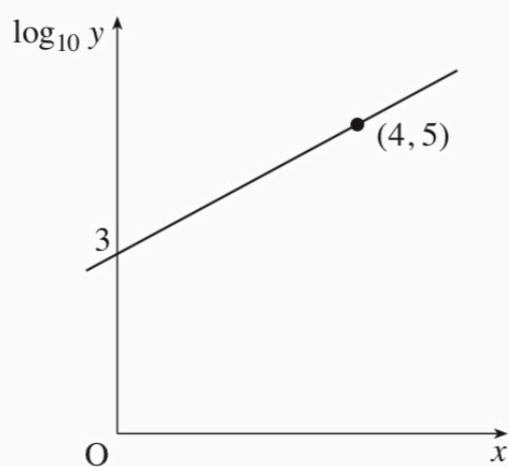**6** Fig. 6 shows the relationship between $\log_{10} x$ and $\log_{10} y$.

Fig. 6

Find y in terms of x. **[5]**

9

Fig. 9

The graph of $\log_{10} y$ against x is a straight line as shown in Fig. 9.

(i) Find the equation for $\log_{10} y$ in terms of x. [3]

(ii) Find the equation for y in terms of x. [2]

12 Answer the whole of this question on the insert provided.

A colony of bats is increasing. The population, P, is modelled by $P = a \times 10^{bt}$, where t is the time in years after 2000.

  (i) Show that, according to this model, the graph of $\log_{10} P$ against t should be a straight line of gradient b. State, in terms of a, the intercept on the vertical axis. [3]

 (ii) The table gives the data for the population from 2001 to 2005.

Year	2001	2002	2003	2004	2005
t	1	2	3	4	5
P	7900	8800	10000	11300	12800

Complete the table of values on the insert, and plot $\log_{10} P$ against t. Draw a line of best fit for the data. [3]

(iii) Use your graph to find the equation for P in terms of t. [4]

(iv) Predict the population in 2008 according to this model. [2]
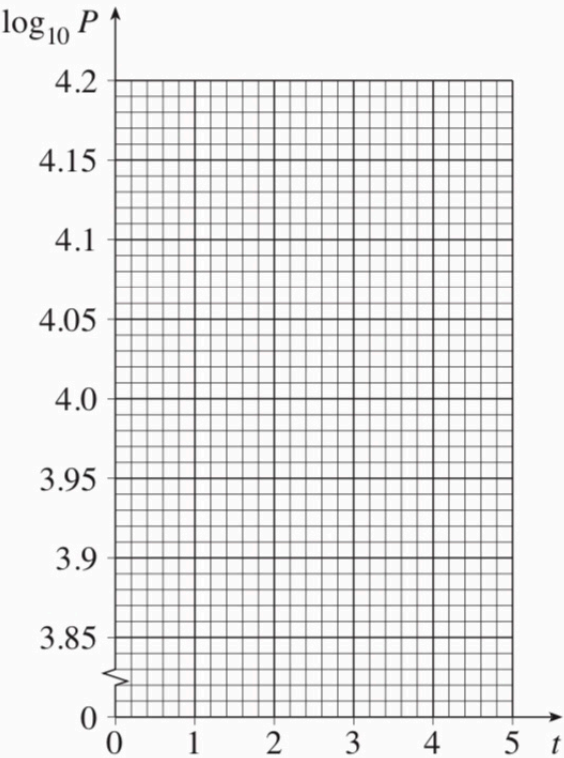
12 (i) ....

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(ii)

Year	2001	2002	2003	2004	2005
t	1	2	3	4	5
P	7900	8800	10000	11300	12800
$\log_{10} P$					

(iii) ....

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(iv) ....

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13 **Answer part (ii) of this question on the insert provided.**

The table gives a firm's monthly profits for the first few months after the start of its business, rounded to the nearest £100.

Number of months after start-up (x)	1	2	3	4	5	6
Profit for this month ($£y$)	500	800	1200	1900	3000	4800

The firm's profits, $£y$, for the xth month after start-up are modelled by

$$y = k \times 10^{ax}$$

where a and k are constants.

(i) Show that, according to this model, a graph of $\log_{10} y$ against x gives a straight line of gradient a and intercept $\log_{10} k$. [2]

(ii) **On the insert**, complete the table and plot $\log_{10} y$ against x, drawing by eye a line of best fit. [3]

(iii) Use your graph to find an equation for y in terms of x for this model. [3]

(iv) For which month after start-up does this model predict profits of about £75 000? [3]

(v) State one way in which this model is unrealistic. [1]

13 (ii)

Number of months after start-up (x)	1	2	3	4	5	6
Profit for this month (£y)	500	800	1200	1900	3000	4800
$\log_{10} y$	2.70					
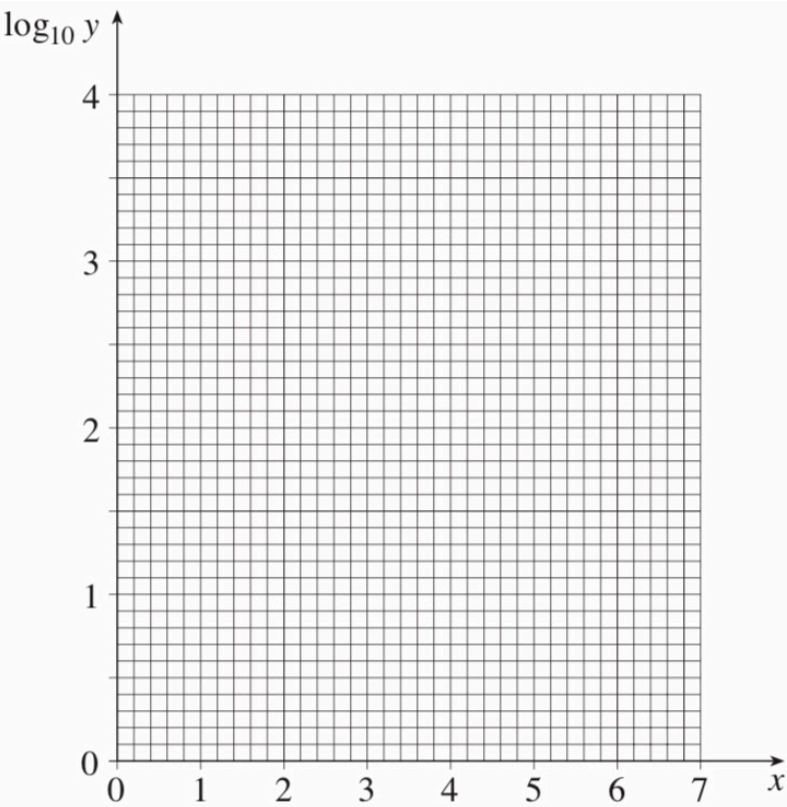
12 **Answer part (ii) of this question on the insert provided.**

The proposal for a major building project was accepted, but actual construction was delayed. Each year a new estimate of the cost was made. The table shows the estimated cost, £y million, of the project t years after the project was first accepted.

Years after proposal accepted (t)	1	2	3	4	5
Cost (£y million)	250	300	360	440	530

The relationship between y and t is modelled by $y = ab^t$, where a and b are constants.

(i) Show that $y = ab^t$ may be written as

$$\log_{10} y = \log_{10} a + t \log_{10} b.$$ [2]

(ii) **On the insert**, complete the table and plot $\log_{10} y$ against t, drawing by eye a line of best fit. [3]

(iii) Use your graph and the results of part (i) to find the values of $\log_{10} a$ and $\log_{10} b$ and hence a and b. [4]

(iv) According to this model, what was the estimated cost of the project when it was first accepted? [1]

(v) Find the value of t given by this model when the estimated cost is £1000 million. Give your answer rounded to 1 decimal place. [2]

12 (ii)

Years after proposal accepted (t)	1	2	3	4	5
Cost (£y million)	250	300	360	440	530
$\log_{10} y$	2.398				
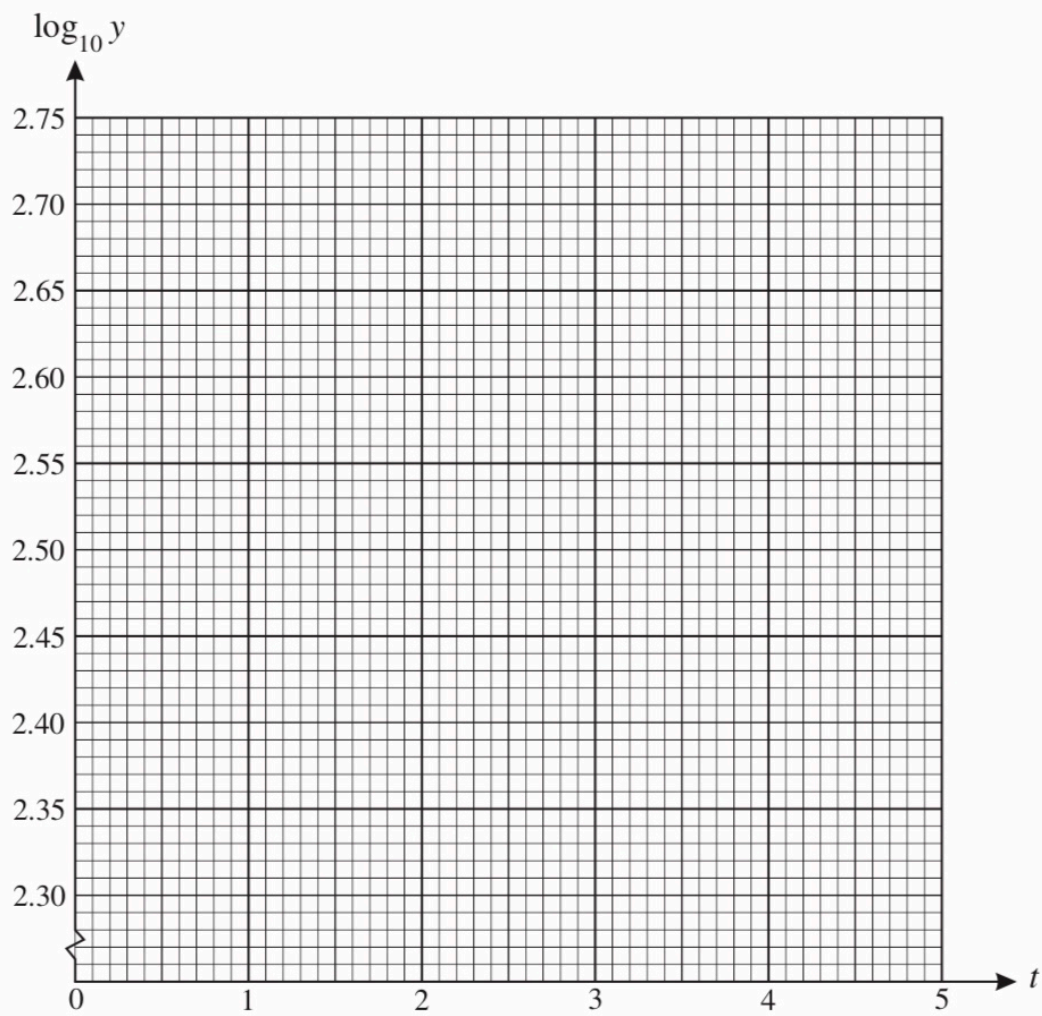
12 Answer part (ii) of this question on the insert provided.

Since 1945 the populations of many countries have been growing. The table shows the estimated population of 15- to 59-year-olds in Africa during the period 1955 to 2005.

Year	1955	1965	1975	1985	1995	2005
Population (millions)	131	161	209	277	372	492

Source: United Nations

Such estimates are used to model future population growth and world needs of resources. One model is $P = a10^{bt}$, where the population is P millions, t is the number of years after 1945 and a and b are constants.

(i) Show that, using this model, the graph of $\log_{10} P$ against t is a straight line of gradient b. State the intercept of this line on the vertical axis. [3]

(ii) **On the insert**, complete the table, giving values correct to 2 decimal places, and plot the graph of $\log_{10} P$ against t. Draw, by eye, a line of best fit on your graph. [3]

(iii) Use your graph to find the equation for P in terms of t. [4]

(iv) Use your results to estimate the population of 15- to 59-year-olds in Africa in 2050. Comment, with a reason, on the reliability of this estimate. [3]

12 (ii)

Year	1955	1965	1975	1985	1995	2005
t	10	20	30	40	50	60
P	131	161	209	277	372	492
$\log_{10} P$	2.12	2.21				
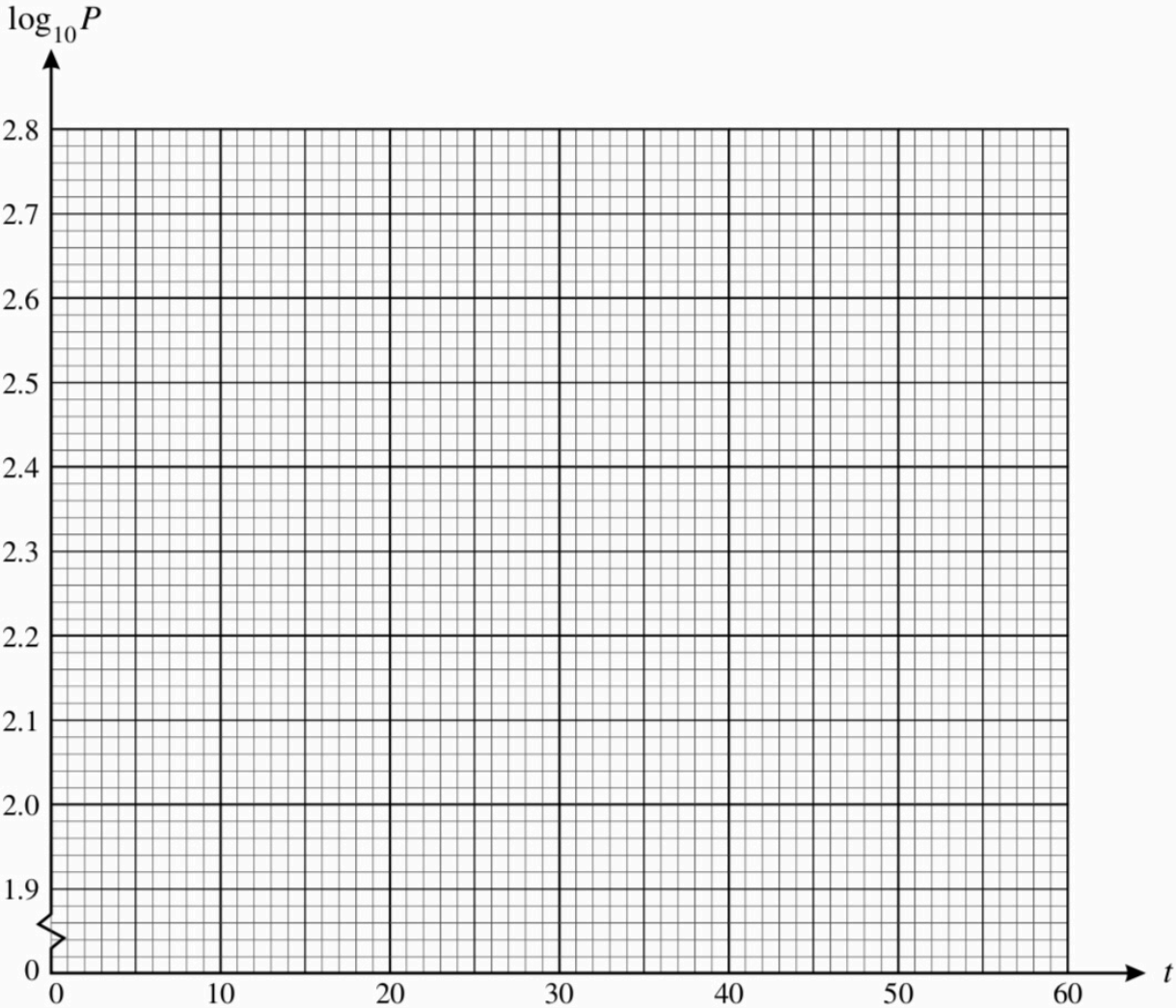
12 The table shows the size of a population of house sparrows from 1980 to 2005.

Year	1980	1985	1990	1995	2000	2005
Population	25 000	22 000	18 750	16 250	13 500	12 000

The 'red alert' category for birds is used when a population has decreased by at least 50% in the previous 25 years.

  (i) Show that the information for this population is consistent with the house sparrow being on red alert in 2005.   **[1]**

The size of the population may be modelled by a function of the form $P = a \times 10^{-kt}$, where P is the population, t is the number of years after 1980, and a and k are constants.

  (ii) Write the equation $P = a \times 10^{-kt}$ in logarithmic form using base 10, giving your answer as simply as possible.   **[2]**

  (iii) Complete the table and draw the graph of $\log_{10} P$ against t, drawing a line of best fit by eye.   **[3]**

  (iv) Use your graph to find the values of a and k and hence the equation for P in terms of t.   **[4]**

  (v) Find the size of the population in 2015 as predicted by this model.

       Would the house sparrow still be on red alert? Give a reason for your answer.   **[3]**

12 (iii)

Year	1980	1985	1990	1995	2000	2005
Population (P)	25 000	22 000	18 750	16 250	13 500	12 000
Years after 1980 (t)	0	5	10	15	20	25
$\log_{10} P$	4.40	4.34				

12 (iii) (continued)