Exercise 7F

1 b) Prove $\frac{\sin B}{\sin A} - \frac{\cos B}{\cos A} \equiv 2 \operatorname{cosec} 2A \sin(B-A)$

$$\frac{\sin B}{\sin A} - \frac{\cos B}{\cos A} = \frac{\sin B\cos A - \cos B \sin A}{\sin A \cos A}$$

$$= \frac{2(\sin B \cos A - \cos B \sin A)}{2 \sin A \cos A}$$

$$= \frac{2\sin(B-A)}{\sin 2A}$$

$$= 2 \operatorname{cosec} 2A \sin(B-A)$$

$\operatorname{cosec} 2A = \frac{1}{\sin 2A}$

$\sin(B-A)$
$= \sin B \cos A - \cos B \sin A$
Jottings

---

1 d) Prove $\frac{\sec^2\theta}{1-\tan^2\theta} \equiv \sec 2\theta$

$\sec 2\theta = \frac{1}{\cos 2\theta}$
$= \frac{1}{\cos^2\theta - \sin^2\theta}$

$$\frac{\sec^2\theta}{1-\tan^2\theta} \equiv \frac{\frac{1}{\cos^2\theta}}{1 - \frac{\sin^2\theta}{\cos^2\theta}}$$

$$\equiv \frac{\frac{1}{\cos^2\theta}}{\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}} \equiv \frac{1}{\cos^2\theta - \sin^2\theta} \equiv \frac{1}{\cos 2\theta} \equiv \sec 2\theta$$

---

1 f) Prove $\frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} \equiv 2$

$$\frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} = \frac{\sin 3\theta\cos\theta - \cos 3\theta\sin\theta}{\sin\theta\cos\theta}$$

$$\equiv \frac{\sin(3\theta - \theta)}{\frac{1}{2}\sin 2\theta}$$

$$\equiv \frac{\sin 2\theta}{\frac{1}{2}\sin 2\theta} \equiv 2$$

1h) Prove $\dfrac{\sec\theta - 1}{\sec\theta + 1} \equiv \tan^2\frac{\theta}{2}$

$$\frac{\sec\theta - 1}{\sec\theta + 1} \equiv \frac{\frac{1}{\cos\theta} - 1}{\frac{1}{\cos\theta} + 1} \equiv \frac{1 - \cos\theta}{1 + \cos\theta}$$

$$\equiv \frac{1 - (1 - 2\sin^2\frac{\theta}{2})}{1 + (2\cos^2\frac{\theta}{2} - 1)}$$

$$\equiv \frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}$$

$$\equiv \tan^2\frac{\theta}{2}$$

$1 + \tan^2\frac{\theta}{2} = \sec^2\frac{\theta}{2}$

$\dfrac{\sin^2\frac{\theta}{2}}{\cos^2\frac{\theta}{2}}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= 1 - 2\sin^2\theta$
$= 2\cos^2\theta - 1$

Prove

2 a) $\sin(A + 60^\circ) + \sin(A - 60^\circ) \equiv \sin A$

$\equiv \sin A\cos 60 + \cos A\sin 60 + \sin A\cos 60 - \cos A\sin 60$

$\frac{1}{2}\sin A + \frac{1}{2}\sin A \equiv \sin A$

Hwk – hand in Monday

Exercise 7F Q1 a, c, e, g, i