1) Newton's 2nd Law of Motion   N2L
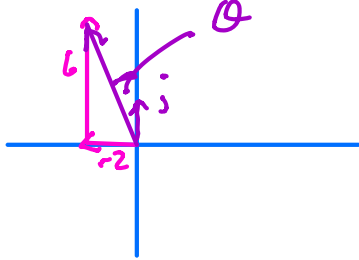
$$\underline{F} = m\underline{a}$$

$$\underline{F_1} = -3\underline{i} + 7\underline{j} = \begin{pmatrix} -3 \\ 7 \end{pmatrix}$$

$$\underline{F_2} = \underline{i} - \underline{j} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

a) $\underline{R} = \underline{F_1} + \underline{F_2} = \begin{pmatrix} -3 \\ 7 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -2 \\ 6 \end{pmatrix}$ N

$$|\underline{R}| = \sqrt{(-2)^2 + 6^2} = \sqrt{40} = 6.32 \text{ N}$$

b)

$$\theta = \tan^{-1}\left(\frac{2}{6}\right)$$

$$\theta = 18.4°$$

3)
a) $\underline{V} = \begin{pmatrix} 4 \\ 9 \end{pmatrix}$

$$|\underline{V}| = \sqrt{4^2 + 9^2}$$
$$= 9.85 \text{ ms}^{-1}$$

b) $\underline{S} = \underline{u}t + \frac{1}{2}\underline{a}t^2$

$$\underline{S} = \begin{pmatrix} 4 \\ 9 \end{pmatrix} \times 6 + \underline{0} = \begin{pmatrix} 24 \\ 54 \end{pmatrix}$$

$$\text{Distance} = \sqrt{24^2 + 54^2} = 59.1 \text{ m}$$

c) Not realistic for large t. Ball will slow down and stop due to friction, air resistance

5) $5\underline{a} + k\underline{b}$ parallel to $8\underline{a} + 2\underline{b}$
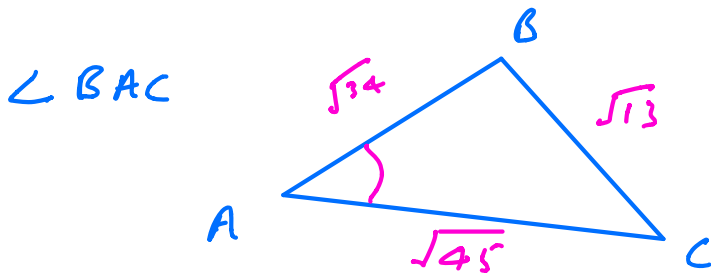
$$k \times \frac{8}{5} = 2$$
$$8k = 10$$
$$k = \frac{10}{8} = 1.25$$

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2) $\overrightarrow{AB} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} \qquad \overrightarrow{AC} \begin{pmatrix} 6 \\ 3 \end{pmatrix}$

a)
$$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$$
$$= \begin{pmatrix} -3 \\ -5 \end{pmatrix} + \begin{pmatrix} 6 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} = 3\underline{i} - 2\underline{j}$$

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b) $\angle BAC$

$$|AB| = \sqrt{3^2 + 5^2} = \sqrt{34}$$
$$|AC| = \sqrt{6^2 + 3^2} = \sqrt{45}$$
$$|BC| = \sqrt{3^2 + (-2)^2} = \sqrt{13}$$

$$\cos A = \frac{\sqrt{34}^2 + \sqrt{45}^2 - \sqrt{13}^2}{2 \times \sqrt{34} \times \sqrt{45}} = \frac{66}{2\sqrt{1530}}$$

$$\cos A = \frac{33}{\sqrt{1530}} = 32.5°$$

$$\angle BAC = 32.5°$$

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c)
$$\text{Area} = \frac{1}{2} bc \sin A$$
$$= \frac{1}{2} \times \sqrt{34} \times \sqrt{45} \, \sin 32.471°$$
$$= 10.5 \text{ units}^2$$

Hwk   Mixed Exercise 11 — Even Numbers

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8. [*In this question, the unit vectors* **i** *and* **j** *are horizontal vectors due east and north respectively.*]

At time $t = 0$, a football player kicks a ball from the point A with position vector $(2\mathbf{i} + \mathbf{j})$ m on a horizontal football field. The motion of the ball is modelled as that of a particle moving horizontally with constant velocity $(5\mathbf{i} + 8\mathbf{j})$ m s^{-1}. Find

(a) the speed of the ball,

(2)

(b) the position vector of the ball after t seconds.

(2)

The point B on the field has position vector $(10\mathbf{i} + 7\mathbf{j})$ m.

(c) Find the time when the ball is due north of B.

(2)

At time $t = 0$, another player starts running due north from B and moves with constant speed v m s^{-1}. Given that he intercepts the ball,

(d) find the value of v.

(6)

(e) State one physical factor, other than air resistance, which would be needed in a refinement of the model of the ball's motion to make the model more realistic.

(1)

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a) $\underline{v} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$ $|\underline{v}| = \sqrt{5^2 + 8^2}$

$= 9.43 \text{ m s}^{-1}$

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b) $\underline{s} - \underline{s_0} = \underline{v}t + \tfrac{1}{2}\underline{a}t^2$

$\underline{s} - \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}t + \underline{0}$

$\underline{s} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}t + \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$\underline{s} = (5t + 2)\underline{i} + (8t + 1)\underline{j}$

c) when ball North of B i component $= 10$

$$\Rightarrow \quad 5t + 2 = 10$$
$$5t = 10 - 2$$
$$5t = 8 \qquad\qquad t = \frac{8}{5} = 1.6 \, s$$

d)   At $t = 1.6$

$$\underline{S} = 10i + (8 \times 1.6 + 1)j$$
$$= 10\underline{i} + 13.8j$$

Runner must travel $(13.8 - 7)$ m in 1.6 s

$$6.8 \text{ m in } 1.6 s$$

$$\text{Speed } v = \frac{6.8}{1.6} = 4.25 \, ms^{-1}$$

e)   Consider friction