Cubics and Quartics

Exercise 1F

1) $f(z) = z^3 - 6z^2 + 21z - 26$

$$f(2) = 2^3 - 6(2)^2 + 21(2) - 26$$
$$= 8 - 24 + 42 - 26$$
$$= 0$$

By factor theorem $(z-2)$ is a factor

$$\begin{array}{r|l} & z^2 - 4z + 13 \\ \hline z-2 & z^3 - 6z^2 + 21z - 26 \\ & \underline{z^3 - 2z^2} \\ & \quad -4z^2 + 21z \\ & \quad \underline{-4z^2 + 8z} \\ & \qquad +13z - 26 \\ & \qquad \underline{+13z - 26} \end{array}$$

$(z-2)(z^2-4z+13)$

$$f(z) = (z-2)(z^2 - 4z + 13)$$

$$z = \frac{4 \pm \sqrt{(-4)^2 - 4 \times 1 \times 13}}{2}$$

$$z = \frac{4 \pm 6i}{2} \qquad z = 2 \pm 3i$$

Roots of $f(z) = 0$ are $z = 2$

$z = 2 + 3i$

$z = 2 - 3i$

Suppose $z^3 - 6z^2 + 21z - 26 = 0$

and that $2+3i$ is a root

Find other 2 roots.

α, β, γ $\qquad \Sigma\alpha = \alpha + \beta + \gamma$

$2+3i \quad 2-3i$ $\qquad = -\frac{-6}{1} = 6$

$2 + \cancel{3i} + 2 - \cancel{3i} + \gamma = 6$

$\underline{\gamma = 2}$

OR

$(z - (2+3i))(z - (2-3i))$

$= ((z-2) - 3i)((z-2) + 3i)$

$= (z-2)^2 + 3^2$

$= z^2 - 4z + 4 + 9$

$= z^2 - 4z + 13$

$(z^2 - 4z + 13)(z - 2) = 0$

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3) $g(z) = 2z^3 - 4z^2 - 5z - 3$

$z = 3$ is a root $\Rightarrow$ $z-3$ is a factor of $g(z)$

$$
\begin{array}{r|l}
 & 2z^2 + 2z + 1 \\
\hline
z-3 & 2z^3 - 4z^2 - 5z - 3 \\
 & \underline{2z^3 - 6z^2} \\
 & \quad +2z^2 - 5z \\
 & \quad \underline{+2z^2 - 6z} \\
 & \qquad\quad +z - 3 \\
 & \qquad\quad \underline{+z - 3}
\end{array}
$$

$$g(z) = (z-3)(2z^2+2z+1)$$

when $2z^2 + 2z + 1 = 0$

$$z = \frac{-2 \pm \sqrt{4-8}}{4} = \frac{-2 \pm 2i}{4} = \frac{1 \pm i}{2}$$

Roots are $z = 3$, $z = \frac{1+i}{2}$, $z = \frac{1-i}{2}$

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11) $g(z) = z^4 + 2z^3 - z^2 + 38z + 130$

$g(2+3i) = 0 \Rightarrow$ $2+3i$ is a root

$2-3i$ is a root

$(z - (2+3i))(z - (2-3i))$

$((z-2) - 3i)((z-2) + 3i)$

$= (z-2)^2 + 3^2 = z^2 - 4z + 4 + 9$

$$= z^2 - 4z + 13$$

$$
\begin{array}{r|l}
 & z^2 + 6z + 10 \\
\hline
z^2 - 4z + 13 & z^4 + 2z^3 - z^2 + 38z + 130 \\
 & \underline{z^4 - 4z^3 + 13z^2} \\
 & \quad 6z^3 - 14z^2 + 38z \\
 & \quad \underline{6z^3 - 24z^2 + 78z} \\
 & \qquad +10z^2 - 40z + 130 \\
 & \qquad \underline{+10z^2 - 40z + 130}
\end{array}
$$

$$g(z) = (z - (2+3i))(z - (2-3i))(z^2 + 6z + 10)$$

$$z = \frac{-6 \pm \sqrt{36 - 40}}{2} = \frac{-6 \pm 2i}{2}$$

$$z = -3 \pm i$$

Roots are $2+3i,\ 2-3i,\ -3+i,\ -3-i$

Homework Exercise 1F - odd numbers