

# Binomial Expansion For Negative And Fractional Powers

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1 \cdot 2} x^2 + \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3} x^3 + \dots$$

Ex 1

$$(1+x)^{-1} = 1 + -1 \cdot x + \frac{-1 \cdot -2}{1 \cdot 2} x^2 + \frac{-1 \cdot -2 \cdot -3}{1 \cdot 2 \cdot 3} x^3 + \dots$$

$$\approx 1 - x + x^2 - x^3 + \dots$$

Ex 2

$$\sqrt{1+x} = (1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x + \frac{\frac{1}{2} \cdot -\frac{1}{2}}{1 \cdot 2} x^2 + \frac{\frac{1}{2} \cdot -\frac{1}{2} \cdot -\frac{3}{2}}{1 \cdot 2 \cdot 3} x^3 + \dots$$

$$= 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \dots$$

Ex 3

$$\sqrt[3]{1-x} = (1+(-x))^{\frac{1}{3}} = 1 + \frac{1}{3}(-x) + \frac{\frac{1}{3} \cdot -\frac{2}{3}}{1 \cdot 2} (-x)^2 + \frac{\frac{1}{3} \cdot -\frac{2}{3} \cdot -\frac{5}{3}}{1 \cdot 2 \cdot 3} (-x)^3 + \dots$$

$$\approx 1 - \frac{1}{3}x - \frac{1}{9}x^2 - \frac{5}{81}x^3 - \dots$$

Ex 4

$$(1+2x)^{\frac{1}{4}} = 1 + \frac{1}{4}(2x) + \frac{\frac{1}{4} \cdot -\frac{3}{4}}{1 \cdot 2} (2x)^2 + \frac{\frac{1}{4} \cdot -\frac{3}{4} \cdot -\frac{7}{4}}{1 \cdot 2 \cdot 3} (2x)^3 + \dots$$

$$= 1 + \frac{1}{2}x - \frac{3}{8}x^2 + \frac{7}{16}x^3$$


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Ex 4A If  $(1+x)^{-\frac{3}{2}}$

$$1 + -\frac{3}{2}x + \frac{-\frac{3}{2} \cdot -\frac{5}{2}}{1 \cdot 2} x^2 + \frac{-\frac{3}{2} \cdot -\frac{5}{2} \cdot -\frac{7}{2}}{1 \cdot 2 \cdot 3} x^3$$

$$1 - \frac{3}{2}x + \frac{15}{8}x^2 - \frac{35}{16}x^3$$

Valid for  $|x| < 1$  or  $-1 < x < 1$

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2c  $(1+2x)^{3/4}$

$$= 1 + \frac{3}{4}(2x) + \frac{\frac{3}{4} \cdot -\frac{1}{4}}{1 \cdot 2} (2x)^2 + \frac{\frac{3}{4} \cdot -\frac{1}{4} \cdot -\frac{5}{4}}{1 \cdot 2 \cdot 3} (2x)^3$$

$$= 1 + \frac{3}{2}x - \frac{3}{8}x^2 + \frac{5}{16}x^3$$

Valid for  $|2x| < 1$

$$|x| < \frac{1}{2} \quad -\frac{1}{2} < x < \frac{1}{2}$$


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Homework to be emailed by Thursday

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