# Homework Solutions

**Q1.**

A manufacturer carried out a survey of the defects in their soft toys. It is found that the probability of a toy having poor stitching is 0.03 and that a toy with poor stitching has a probability of 0.7 of splitting open. A toy without poor stitching has a probability of 0.02 of splitting open.

(a) Draw a tree diagram to represent this information.

**(3)**

(b) Find the probability that a randomly chosen soft toy has exactly one of the two defects, poor stitching or splitting open.

**(3)**

The manufacturer also finds that soft toys can become faded with probability 0.05 and that this defect is independent of poor stitching or splitting open. A soft toy is chosen at random.

(c) Find the probability that the soft toy has none of these 3 defects.

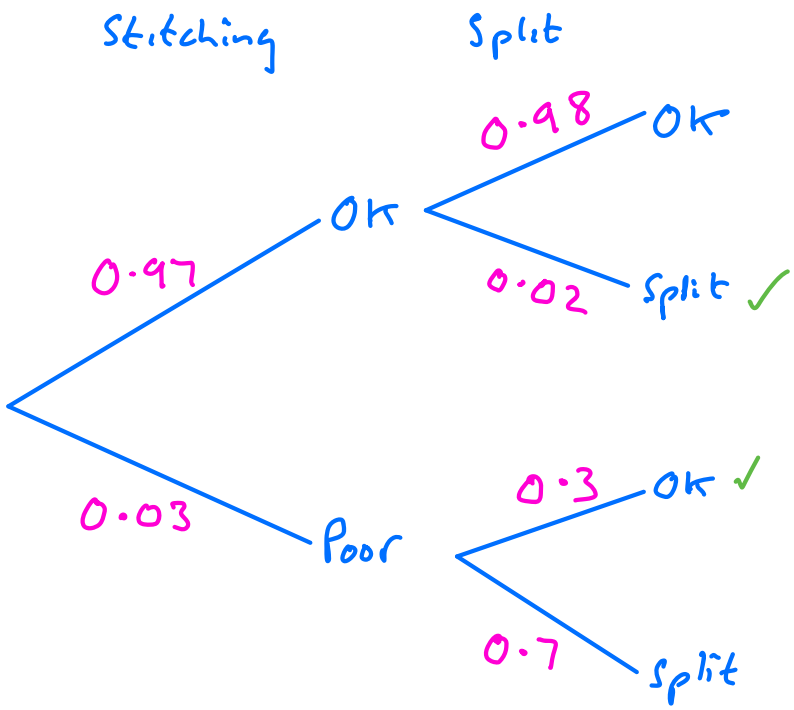
**(2)**

(d) Find the probability that the soft toy has exactly one of these 3 defects.

**(4)**

**(Total 12 marks)**

a)




b) $P(\text{Exactly one defect}) = 0.97 \times 0.02 + 0.03 \times 0.3$

$= 0.0284$

c) $P(\text{No defects}) = 0.97 \times 0.98 \times 0.95$

$= 0.90307$

d) P(Exactly 1 defect)

$= 0.0284 \times 0.95 + 0.97 \times 0.98 \times 0.05$

$= 0.07451$

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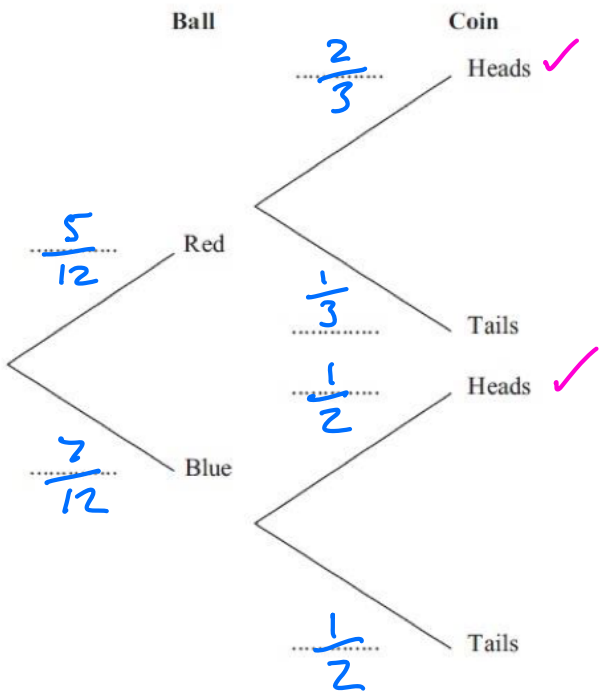
**Q3.**

An experiment consists of selecting a ball from a bag and spinning a coin. The bag contains 5 red balls and 7 blue balls. A ball is selected at random from the bag, its colour is noted and then the ball is returned to the bag.

When a red ball is selected, a biased coin with probability $\frac{2}{3}$ of landing heads is spun. When a blue ball is selected a fair coin is spun.

(a) Complete the tree diagram below to show the possible outcomes and associated probabilities.




(2)

Shivani selects a ball and spins the appropriate coin.

(b) Find the probability that she obtains a head.

(2)

$P(\text{Head}) = \frac{5}{12} \times \frac{2}{3} + \frac{7}{12} \times \frac{1}{2} = \frac{41}{72}$

Given that Tom selected a ball at random and obtained a head when he spun the appropriate coin,

(c) find the probability that Tom selected a red ball.

**(3)**

Shivani and Tom each repeat this experiment.

(d) Find the probability that the colour of the ball Shivani selects is the same as the colour of the ball Tom selects.

**(3)**

**(Total 10 marks)**

c) $P(\text{Red} \mid \text{Head}) = \dfrac{P(\text{Red} \cap \text{Head})}{P(\text{Head})}$

$= \dfrac{\frac{5}{12} \times \frac{2}{3}}{\frac{41}{72}} = \dfrac{\frac{10}{36}}{\frac{41}{72}} = \frac{10}{36} \times \frac{72}{41}$

$= \frac{20}{41}$

d) RR or BB

$\frac{5}{12} \times \frac{5}{12} + \frac{7}{12} \times \frac{7}{12} = \frac{25+49}{144} = = \frac{37}{72}$

**Q5.**

On a randomly chosen day the probability that Bill travels to school by car, by bicycle or on foot is $\frac{1}{2}$, $\frac{1}{6}$ and $\frac{1}{3}$ respectively. The probability of being late when using these methods of travel is $\frac{1}{5}$, $\frac{2}{5}$ and $\frac{1}{10}$ respectively.

(a) Draw a tree diagram to represent this information.

**(3)**

(b) Find the probability that on a randomly chosen day
(i) Bill travels by foot and is late,
(ii) Bill is not late.

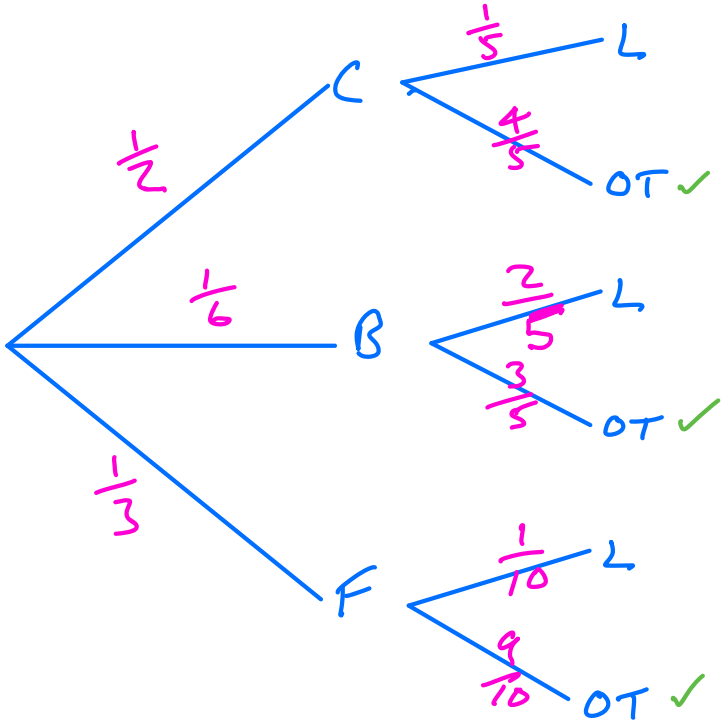
**(4)**

(c) Given that Bill is late, find the probability that he did not travel on foot.

**(4)**

**(Total 11 marks)**

a)




b) i) $P(F \cap L) = \frac{1}{3} \times \frac{1}{10} = \frac{1}{30}$

ii) $P(\text{On time}) = \frac{1}{2} \times \frac{4}{5} + \frac{1}{6} \times \frac{3}{5} + \frac{1}{3} \times \frac{9}{10}$

$= \frac{4}{10} + \frac{3}{30} + \frac{9}{30} = \frac{8}{10}$

c) $P(F' \mid L)$ $\quad P(\text{Late}) = 1 - \frac{8}{10} = \frac{2}{10}$

$P(F' \cap L) = \frac{1}{2} \times \frac{1}{5} + \frac{1}{6} \times \frac{2}{5} = \frac{1}{6}$

$P(F' \mid L) = \frac{P(F' \cap L)}{P(L)} = \frac{\frac{1}{6}}{\frac{2}{10}} = \frac{1}{6} \times \frac{10}{2} = \frac{5}{6}$

**Q7.**

In a large company,

78% of employees are car owners,
30% of these car owners are also bike owners,
85% of those who are not car owners are bike owners.

(a) Draw a tree diagram to represent this information.

**(3)**

An employee is selected at random.

(b) Find the probability that the employee is a car owner or a bike owner but not both.

**(2)**

Another employee is selected at random.

Given that this employee is a bike owner,

(c) find the probability that the employee is a car owner.

**(3)**

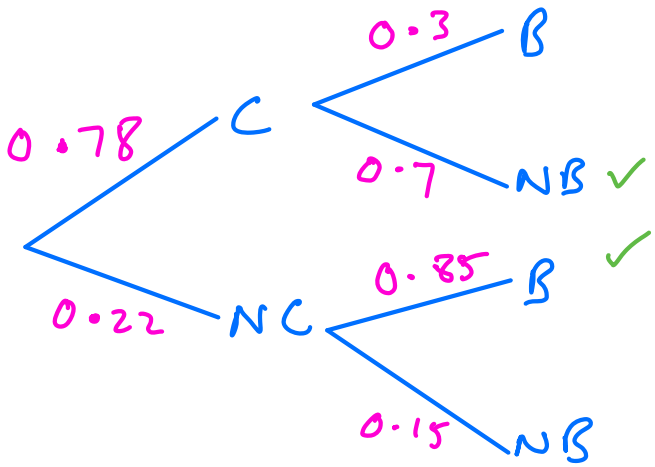
Two employees are selected at random.

(d) Find the probability that only one of them is a bike owner.

**(3)**

**(Total 11 marks)**

a)




b) $P(\text{Owns one of car/bike}) = 0.78 \times 0.7 + 0.22 \times 0.85 = 0.733$

c) $P(C \mid B) = \dfrac{P(C \cap B)}{P(B)}$

$= \dfrac{0.78 \times 0.3}{0.78 \times 0.3 + 0.22 \times 0.85} = \dfrac{234}{421} = 0.5558$

d) $P(B, NB) + P(NB, B)$

$P(B) = 0.78 \times 0.3 + 0.22 \times 0.85 = 0.421$

$P(NB) = 1 - 0.421 = 0.579$

$P(B, NB) + P(NB, B)$

$= 0.421 \times 0.579 \times 2 = 0.487518$

$= 0.488$ to 3 s.f.

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