

Question Number	Scheme	Marks
Q2 (a) $z^3 = 4\sqrt{2} - 4\sqrt{2}i$, $-\pi < \theta \leq \pi$		
	$r = \sqrt{(4\sqrt{2})^2 + (-4\sqrt{2})^2} = \sqrt{32 + 32} = \sqrt{64} = 8$ $\theta = -\tan^{-1}\left(\frac{4\sqrt{2}}{4\sqrt{2}}\right) = -\frac{\pi}{4}$	A valid attempt to find the modulus and argument of $4\sqrt{2} - 4\sqrt{2}i$. M1
	$z^3 = 8\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$	
	So, $z = (8)^{\frac{1}{3}}\left(\cos\left(\frac{-\frac{\pi}{4}}{3}\right) + i\sin\left(\frac{-\frac{\pi}{4}}{3}\right)\right)$	Taking the cube root of the modulus and dividing the argument by 3. M1
	$\Rightarrow z = 2\left(\cos\left(-\frac{\pi}{12}\right) + i\sin\left(-\frac{\pi}{12}\right)\right)$	$2\left(\cos\left(-\frac{\pi}{12}\right) + i\sin\left(-\frac{\pi}{12}\right)\right)$ A1
	Also, $z^3 = 8\left(\cos\left(\frac{7\pi}{4}\right) + i\sin\left(\frac{7\pi}{4}\right)\right)$ or $z^3 = 8\left(\cos\left(-\frac{9\pi}{4}\right) + i\sin\left(-\frac{9\pi}{4}\right)\right)$	Adding or subtracting 2π to the argument for z^3 in order to find other roots. M1
	$\Rightarrow z = 2\left(\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12}\right)$ and $z = 2\left(\cos\left(\frac{-3\pi}{4}\right) + i\sin\left(\frac{-3\pi}{4}\right)\right)$	Any one of the final two roots A1 Both of the final two roots. A1
	Special Case 1: Award SC: M1M1A1M1A0A0 for ALL three of $2\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$, $2\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$ and $2\left(\cos\left(\frac{-7\pi}{12}\right) + i\sin\left(\frac{-7\pi}{12}\right)\right)$.	[6]
	Special Case 2: If r is incorrect (and not equal to 8) and candidate states the brackets () correctly then give the first accuracy mark ONLY where this is applicable.	

Question Number	Scheme	Marks
4(a)	Modulus = 16 Argument = $\arctan(-\sqrt{3}) = \frac{2\pi}{3}$	B1 M1A1 (3)
(b)	$z^3 = 16^3(\cos(\frac{2\pi}{3}) + i \sin(\frac{2\pi}{3}))^3 = 16^3(\cos 2\pi + i \sin 2\pi) = 4096$ or 16^3	M1 A1 (2)
(c)	$w = 16^{\frac{1}{4}}(\cos(\frac{2\pi}{3}) + i \sin(\frac{2\pi}{3}))^{\frac{1}{4}} = 2(\cos(\frac{\pi}{6}) + i \sin(\frac{\pi}{6})) \quad (= \sqrt{3} + i)$ OR $-1 + \sqrt{3}i$ OR $-\sqrt{3} - i$ OR $1 - \sqrt{3}i$	<input type="checkbox"/> M1 A1ft <input type="checkbox"/> M1A2(1,0) (5)
		10

Question Number	Scheme	Marks
3.		
(a)	$r = \sqrt{(-2)^2 + (2\sqrt{3})^2} = 4$ $\tan \theta = -\sqrt{3}$ (Also allow M mark for $\tan \theta = \sqrt{3}$) M mark can be implied by $\theta = \pm \frac{2\pi}{3}$ or $\theta = \pm \frac{\pi}{3}$ $\theta = \frac{2\pi}{3}$	B1 M1 A1 (3)
(b)	Finding the 4 th root of their r : $r = 4^{\frac{1}{4}} (= \sqrt{2})$ For one root, dividing their θ by 4: $\theta = \frac{2\pi}{3} \div 4 = \frac{\pi}{6}$ For another root, add or subtract a multiple of 2π to their θ and divide by 4 in correct order. $\sqrt{2}(\cos \theta + i \sin \theta)$, where $\theta = -\frac{5\pi}{6}, -\frac{\pi}{3}, \frac{\pi}{6}, \frac{2\pi}{3}$	M1 M1 M1 A1 A1 (5) 8
	Notes (a) M1 Accept $\pm \sqrt{3}$ or $\pm \frac{1}{\sqrt{3}}$ (b) A1 Accept awrt 2.1. A0 if in degrees. 2^{nd} M1 for awrt 0.52 1^{st} A1 for two correct values 2^{nd} A1 for all correct values in correct form and no more	

Question Number	Scheme	Marks
2		
(a)	$z = 5\sqrt{3} - 5i = r(\cos \theta + i \sin \theta)$ $r = \sqrt{(5^2 \times 3 + 5^2)} = 10$	B1 (1)
(b)	$\arg z = \arctan\left(-\frac{5}{5\sqrt{3}}\right) = -\frac{\pi}{6}$ (or $-\frac{\pi}{6} \pm 2n\pi$)	M1A1 (2)
(c)	$\left \frac{w}{z}\right = \frac{2}{10} = \frac{1}{5}$ or 0.2	B1 (1)
(d)	$\arg\left(\frac{w}{z}\right) = \frac{\pi}{4} - \left(-\frac{\pi}{6}\right), = \frac{5\pi}{12}$ (or $\frac{5\pi}{12} \pm 2n\pi$)	M1,A1 (2) [6]

Notes for Question 2

(a)

B1 for $|z|=10$ no working needed

(b)

M1 for $\arg z = \arctan\left(\pm \frac{5}{5\sqrt{3}}\right)$, $\tan(\arg z) = \pm \frac{5}{5\sqrt{3}}$, $\arg z = \arctan\left(\pm \frac{5\sqrt{3}}{5}\right)$ or
 $\tan(\arg z) = \pm \frac{5\sqrt{3}}{5}$ OR use their $|z|$ with sin or cos used correctly

A1 for $= -\frac{\pi}{6}$ (or $-\frac{\pi}{6} \pm 2n\pi$) (must be 4th quadrant)

(c)

B1 for $\left|\frac{w}{z}\right| = \frac{2}{10}$ or $\frac{1}{5}$ or 0.2

(d)

M1 for $\arg\left(\frac{w}{z}\right) = \frac{\pi}{4} - \arg z$ using **their** $\arg z$ A1 for $\frac{5\pi}{12}$ (or $\frac{5\pi}{12} \pm 2n\pi$)*Alternative for (d):*

$$\text{Find } \frac{w}{z} = \frac{(\sqrt{6}-\sqrt{2}) + (\sqrt{6}+\sqrt{2})i}{20}$$

$$\tan\left(\arg \frac{w}{z}\right) = \frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}} \quad \text{M1 from their } \frac{w}{z}$$

$$\arg\left(\frac{w}{z}\right) = \frac{5\pi}{12} \quad \text{A1 cao}$$

Work for (c) and (d) may be seen together – give B and A marks only if modulus and argument are clearly identified

ie $\frac{1}{5}\left(\cos \frac{5\pi}{12} + i \sin \frac{5\pi}{12}\right)$ alone scores B0M1A0

Question Number	Scheme	Marks
4 (a)	<p>Assume true for $n = k$: $z^k = r^k (\cos k\theta + i \sin k\theta)$</p> $\begin{aligned} n = k+1: \quad z^{k+1} &= (z^k \times z) = r^k (\cos k\theta + i \sin k\theta) \times r (\cos \theta + i \sin \theta) \\ &= r^{k+1} (\cos k\theta \cos \theta - \sin k\theta \sin \theta + i(\sin k\theta \cos \theta + \cos k\theta \sin \theta)) \\ &= r^{k+1} (\cos((k+1)\theta) + i \sin((k+1)\theta)) \end{aligned}$ <p>$\therefore \underline{\text{if true for } n = k}, \quad \underline{\text{also true for } n = k+1}$</p> <p>$k = 1 \quad \underline{z^1 = r^1 (\cos \theta + i \sin \theta)}; \quad \underline{\text{True for } n = 1} \quad \therefore \underline{\text{true for all } n}$</p>	M1 M1 M1depA1cso A1cso (5)
(b)	<p><i>Alternative:</i> See notes for use of $r e^{i\theta}$ form</p> $w = 3 \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right)$ $w^5 = 3^5 \left(\cos \frac{15\pi}{4} + i \sin \frac{15\pi}{4} \right)$ $w^5 = 243 \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} i \right) \quad \left[= \frac{243\sqrt{2}}{2} - \frac{243\sqrt{2}}{2} i \quad \text{or} \right] \quad \text{oe}$	M1 A1 (2) [7]

Notes for Question 4

(a)

NB: Allow each mark if $n, n + 1$ used instead of $k, k + 1$

M1 for using the result for $n = k$ to write $z^{k+1} (= z^k \times z) = r^k (\cos k\theta + i \sin k\theta) \times r(\cos \theta + i \sin \theta)$

M1 for multiplying out and collecting real and imaginary parts, using $i^2 = -1$

OR using sum of arguments and product of moduli to get $r^{k+1} (\cos(k\theta + \theta) + i \sin(k\theta + \theta))$

M1dep for using the addition formulae to obtain single cos and sin terms

OR factorise the argument $r^{k+1} (\cos \theta(k+1) + i \sin \theta(k+1))$

Dependent on the second M mark.

A1cso for $r^{k+1} (\cos(k+1)\theta + i \sin(k+1)\theta)$ Only give this mark if all previous steps are fully correct.

A1cso All 5 underlined statements must be seen

Alternative: Using Euler's form

$z = r(\cos \theta + i \sin \theta) = r e^{i\theta}$	M1 May not be seen explicitly
$z^{k+1} = z^k \times z = (r e^{i\theta})^k \times r e^{i\theta} = r^k e^{ik\theta} \times r e^{i\theta}$	M1
$= r^{k+1} e^{i(k+1)\theta}$	M1dep on 2 nd M mark
$= r^{k+1} (\cos(k+1)\theta + i \sin(k+1)\theta)$	A1cso
$k = 1 \quad z^1 = r^1 (\cos \theta + i \sin \theta)$	
True for $n = 1 \therefore$ true for all n etc	A1 cso All 5 underlined statements must be seen

(b)

M1 for attempting to apply de Moivre to w or attempting to expand w^5 and collecting real and imaginary parts, but no need to simplify these.

A1cao for $243 \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i \right) \quad \left[= \frac{243\sqrt{2}}{2} - \frac{243\sqrt{2}}{2}i \right]$ (oe eg 3^5 instead of 243)