Proportion

y varies directly with the square of x

$$y = kx^2$$

y is directly proportional to the square root of x

$$y = k\sqrt{x}$$

y varies inversely with the cube of x

$$y = \frac{k}{x^3}$$

Ex 1 $\quad D \propto x$

$$\begin{cases} D = 36 \\ x = 5 \end{cases}$$

Find D when $x = 8$

$$D = kx$$

$$36 = 5k$$

$$\frac{36}{5} = k$$

$$7.2 = k$$

$$D = 7.2x$$

$x = 8$

$$D = 7.2 \times 8$$

$$D = 57.6$$

Ex 2 $\quad T$ inversely proportional to d^2

$$T = \frac{k}{d^2}$$

$$\begin{cases} T = 12 \\ d = 8 \end{cases}$$

$$12 = \frac{k}{8^2}$$

$$k = 12 \times 8^2$$

$$k = 768$$

$$T = \frac{768}{d^2}$$

when $d = 0.5$ $\quad T = \frac{768}{0.5^2} = 3072$

Surds

Task 1 $\quad \sqrt{2} + \sqrt{50}$

$= \sqrt{2} + \sqrt{25 \times 2}$

$= \sqrt{2} + 5\sqrt{2} \quad = 6\sqrt{2}$

Ex 2 $\quad \sqrt{45} - 2\sqrt{20} + \sqrt{80}$

$\sqrt{9 \times 5} - 2\sqrt{4 \times 5} + \sqrt{16 \times 5}$

$= 3\sqrt{5} - 2 \times 2\sqrt{5} + 4\sqrt{5}$

$= 3\sqrt{5}$

Task 2 $\quad (3 + 2\sqrt{5})(4 - 3\sqrt{5})$

$= 12 + 8\sqrt{5} - 9\sqrt{5} - 30$

$= -18 - \sqrt{5}$

Ex 2 $\quad (4 + 3\sqrt{2})(5 - \sqrt{2})$

$= 20 + 15\sqrt{2} - 4\sqrt{2} - 6$

$= 14 + 11\sqrt{2}$

Task 3

$$\frac{6}{\sqrt{2}}$$ write as $a\sqrt{2}$

$$\frac{6}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$$

$$\frac{3+\sqrt{2}}{5-\sqrt{2}}$$ write as $a + b\sqrt{2}$

$$\frac{3+\sqrt{2}}{5-\sqrt{2}} \times \frac{5+\sqrt{2}}{5+\sqrt{2}} = \frac{(3+\sqrt{2})(5+\sqrt{2})}{5^2-\sqrt{2}^2}$$

$$= \frac{15+5\sqrt{2}+3\sqrt{2}+2}{25-2}$$

$$= \frac{17+8\sqrt{2}}{23}$$

$$= \frac{17}{23} + \frac{8}{23}\sqrt{2}$$

Completing the Square

Ex 1

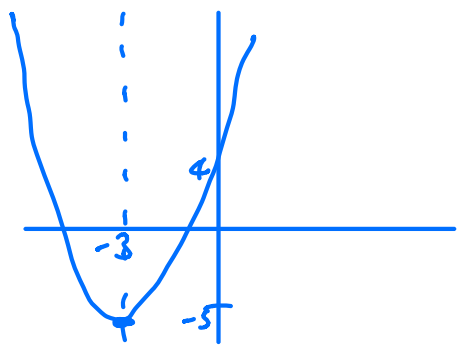
$$y = x^2 + 6x + 4$$

write as $y = (x+a)^2 + b$

$$y = (x+3)^2 + 4 - 9$$

$$y = (x+3)^2 - 5$$

Min pt $(-3, -5)$



$(x+3)(x+3)$

$x^2 + 3x + 3x + 9$

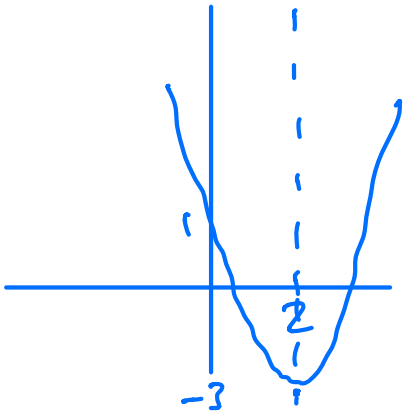
$x^2 + 6x + 9$

Ex 2

$y = x^2 - 4x + 1$

$y = (x-2)^2 + 1 - 4$

$y = (x-2)^2 - 3$



Min pt $(2, -3)$

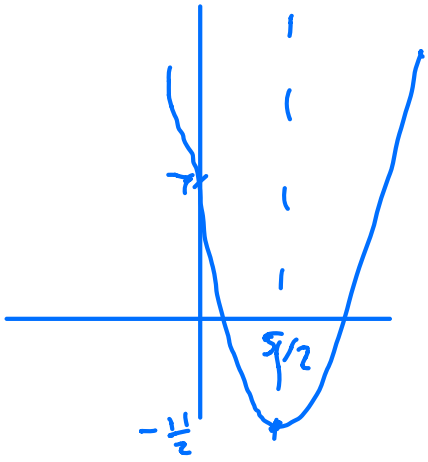
$y = 2x^2 - 10x + 7$

$y = 2\left[x^2 - 5x + \frac{7}{2}\right]$

$y = 2\left[\left(x - \frac{5}{2}\right)^2 + \frac{7}{2} - \frac{25}{4}\right]$

$y = 2\left(x - \frac{5}{2}\right)^2 + 7 - \frac{25}{2}$

$y = 2\left(x - \frac{5}{2}\right)^2 - \frac{11}{2}$



Min pt $\left(5/2, -\frac{11}{2}\right)$