Leave blank

2. $$\mathrm{f}(x) = x^3 + 3x^2 + 4x - 12$$

(a) Show that the equation $\mathrm{f}(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{4(3-x)}{(3+x)}\right)}, \qquad x \neq -3$$

(3)

The equation $x^3 + 3x^2 + 4x - 12 = 0$ has a single root which is between 1 and 2

(b) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{4(3-x_n)}{(3+x_n)}\right)}, \; n \geqslant 0$$

with $x_0 = 1$ to find, to 2 decimal places, the value of x_1, x_2 and x_3.

(3)

The root of $\mathrm{f}(x) = 0$ is α.

(c) By choosing a suitable interval, prove that $\alpha = 1.272$ to 3 decimal places.

(3)

a)

$$x^3 + 3x^2 + 4x - 12 = 0$$

$$x^3 + 3x^2 = 12 - 4x$$

$$x^2(x+3) = 4(3-x)$$

$$x^2 = \frac{4(3-x)}{3+x}$$

$$x = \sqrt{\frac{4(3-x)}{(3+x)}}$$

(b) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{4(3-x_n)}{(3+x_n)}\right)}, \; n \geqslant 0$$

with $x_0 = 1$ to find, to 2 decimal places, the value of x_1, x_2 and x_3.

b)

$$x_1 = \sqrt{\frac{4(3-1)}{(3+1)}} = 1.414 = 1.41 \text{ to 2 d.p}$$

$$x_2 = \sqrt{\frac{4(3-1.414)}{(3+1.414)}} = 1.199 = 1.20 \text{ to 2 d.p.}$$

$$x_3 = \sqrt{\frac{4(3-1.199)}{(3+1.199)}} = 1.310 = 1.31 \text{ to 2 d.p.}$$

The root of $f(x)=0$ is α . $\qquad f(x)=x^3+3x^2+4x-12$

(c) By choosing a suitable interval, prove that $\alpha = 1.272$ to 3 decimal places.

$f(1.2715) = 1.2715^3 + 3(1.2715)^2 + 4(1.2715) - 12 = -8.21 \times 10^{-3} < 0$

$f(1.2725) = 1.2725^3 + 3(1.2725)^2 + 4(1.2725) - 12 = 8.27 \times 10^{-3} > 0$

$f(x)$ is continuous so $f(x)=0$ for some $x \in (1.2715, 1.2725)$

$\therefore$ root is $x = 1.272$ to 3 d.p.
