COMPLETING THE SQUARE — EXERCISE

Exercise A

Write the following functions in the form $y = a(x+p)^2+q$

Identify the turning point at $(-p, q)$ and the line of symmetry $x = -p$. Sketch the graphs.

1. $y = x^2 + 12x + 10$
2. $y = x^2 - 8x - 5$
3. $y = x^2 + 3x + 7$
4. $y = x^2 - 9x - 1$
5. $y = 2x^2 + 8x + 3$
6. $y = 5x^2 - 10x + 8$
7. $y = -x^2 + 7x + 6$
8. $y = -2x^2 - 14x + 9$

Exercise B

Use completing the square to solve the following equations.

1. $x^2 - 6x + 8 = 0$
2. $x^2 - 7x + 12 = 0$
3. $x^2 - 3x - 4 = 0$
4. $x^2 + 3x + 1 = 0$ Answer in surd form
5. $x^2 - 5x + 3 = 0$ Answer in surd form

COMPLETING THE SQUARE — EXERCISE

Exercise A

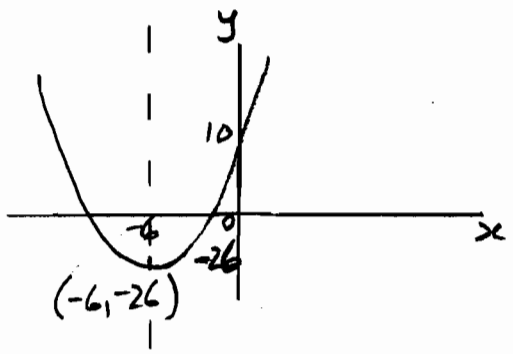
1. $y = x^2 + 12x + 10$

$y = (x+6)^2 + 10 - 36$

$y = (x+6)^2 - 26$

Line of symmetry: $x = -6$

Minimum point: $(-6, -26)$



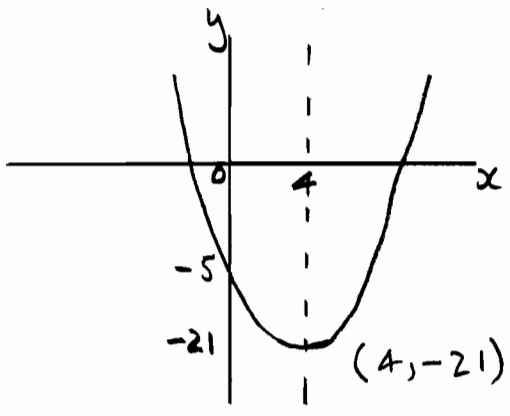
2. $y = x^2 - 8x - 5$

$y = (x-4)^2 - 5 - 16$

$y = (x-4)^2 - 21$

Line of symmetry: $x = 4$

Minimum point: $(4, -21)$



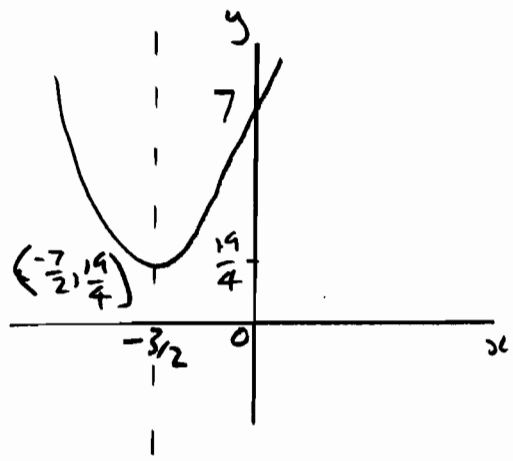
3. $y = x^2 + 3x + 7$

$y = \left(x + \frac{3}{2}\right)^2 + 7 - \frac{9}{4}$

$y = \left(x + \frac{3}{2}\right)^2 + \frac{19}{4}$

Line of symmetry: $x = -\frac{3}{2}$

Minimum point: $\left(-\frac{3}{2}, \frac{19}{4}\right)$



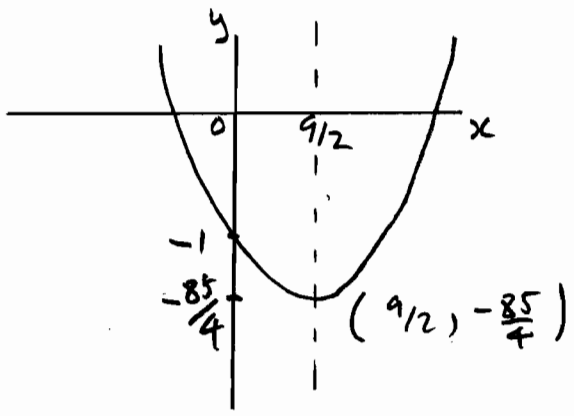
4. $y = x^2 - 9x - 1$

$y = \left(x - \frac{9}{2}\right)^2 - 1 - \frac{81}{4}$

$y = \left(x - 9/2\right)^2 - \frac{85}{4}$

Line of symmetry: $x = \frac{9}{2}$

Minimum point: $\left(\frac{9}{2}, -\frac{85}{4}\right)$



COMPLETING THE SQUARE

EXERCISE

5. $y = 2x^2 + 8x + 3$

$y = 2\left[x^2 + 4x + \frac{3}{2}\right]$

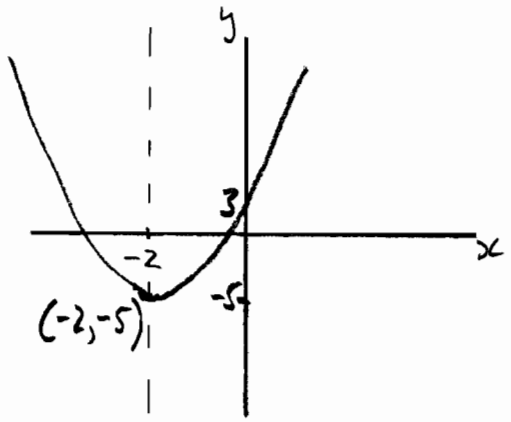
$y = 2\left[(x+2)^2 + \frac{3}{2} - 4\right]$

$y = 2\left[(x+2)^2 - \frac{5}{2}\right]$

$y = 2(x+2)^2 - 5$

Line of symmetry: $x = -2$

Minimum point: $(-2, -5)$



6. $y = 5x^2 - 10x + 8$

$y = 5\left[x^2 - 2x + \frac{8}{5}\right]$

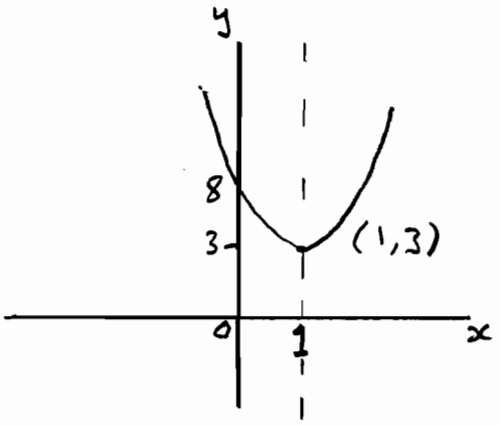
$y = 5\left[(x-1)^2 + \frac{8}{5} - 1\right]$

$y = 5\left[(x-1)^2 + \frac{3}{5}\right]$

$y = 5(x-1)^2 + 3$

Line of symmetry: $x = 1$

Minimum point: $(1, 3)$



7. $y = -x^2 + 7x + 6$

$y = -1\left[x^2 - 7x - 6\right]$

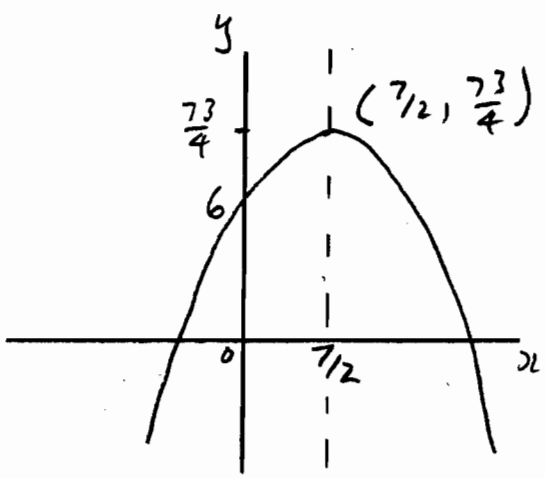
$y = -1\left[(x - \frac{7}{2})^2 - 6 - \frac{49}{4}\right]$

$y = -1\left[(x - \frac{7}{2})^2 - \frac{73}{4}\right]$

$y = -(x - \frac{7}{2})^2 + \frac{73}{4}$

Line of symmetry: $x = \frac{7}{2}$

Maximum point $\left(\frac{7}{2}, \frac{73}{4}\right)$



COMPLETING THE SQUARE

8. $y = -2x^2 - 14x + 9$

$y = -2\left[x^2 + 7x - \frac{9}{2}\right]$

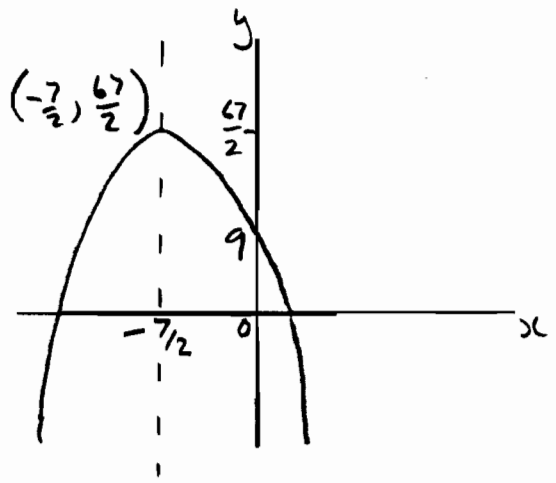
$y = -2\left[\left(x+\frac{7}{2}\right)^2 - \frac{9}{2} - \frac{49}{4}\right]$

$y = -2\left[\left(x+\frac{7}{2}\right)^2 - \frac{67}{4}\right]$

$y = -2\left(x+\frac{7}{2}\right)^2 + \frac{67}{2}$

Line of symmetry: $x = -\frac{7}{2}$

Maximum point $\left(-\frac{7}{2}, \frac{67}{2}\right)$



Exercise B

1. $x^2 - 6x + 8 = 0$

$(x-3)^2 + 8 - 9 = 0$

$(x-3)^2 - 1 = 0$

$(x-3)^2 = 1$

$x - 3 = \pm 1$

$x = +3 \pm 1$

$x = 4$ or $x = 2$

EXERCISE

2. $x^2 - 7x + 12 = 0$

$\left(x - \frac{7}{2}\right)^2 + 12 - \frac{49}{4} = 0$

$\left(x - \frac{7}{2}\right)^2 - \frac{1}{4} = 0$

$\left(x - \frac{7}{2}\right)^2 = \frac{1}{4}$

$x - \frac{7}{2} = \pm \frac{1}{2}$

$x = +\frac{7}{2} \pm \frac{1}{2}$

$x = \frac{8}{2} = 4$ or $x = \frac{6}{2} = 3$

$x = 4$ or $x = 3$

3. $x^2 - 3x - 4 = 0$

$\left(x - \frac{3}{2}\right)^2 - 4 - \frac{9}{4} = 0$

$\left(x - \frac{3}{2}\right)^2 - \frac{25}{4} = 0$

$\left(x - \frac{3}{2}\right)^2 = \frac{25}{4}$

$x - \frac{3}{2} = \pm \frac{5}{2}$

$x = +\frac{3}{2} \pm \frac{5}{2}$

COMPLETING THE SQUARE

EXERCISE

3 cont)

$x = +\frac{3}{2} + \frac{5}{2} = \frac{8}{2} = 4$

or $x = +\frac{3}{2} - \frac{5}{2} = -\frac{2}{2} = -1$

$x = 4$ or $x = -1$

4. $x^2 + 3x + 1 = 0$

$\left(x + \frac{3}{2}\right)^2 + 1 - \frac{9}{4} = 0$

$\left(x + \frac{3}{2}\right)^2 - \frac{5}{4} = 0$

$\left(x + \frac{3}{2}\right)^2 = \frac{5}{4}$

$x + \frac{3}{2} = \pm \frac{\sqrt{5}}{2}$

$x = -\frac{3}{2} \pm \frac{\sqrt{5}}{2}$

$\therefore \quad x = -\frac{3}{2} + \frac{\sqrt{5}}{2}$

or $x = -\frac{3}{2} - \frac{\sqrt{5}}{2}$

5. $x^2 - 5x + 3 = 0$

$\left(x - \frac{5}{2}\right)^2 + 3 - \frac{25}{4} = 0$

$\left(x - \frac{5}{2}\right)^2 - \frac{13}{4} = 0$

$\left(x - \frac{5}{2}\right)^2 = \frac{13}{4}$

$x - \frac{5}{2} = \pm \frac{\sqrt{13}}{2}$

$x = +\frac{5}{2} \pm \frac{\sqrt{13}}{2}$

$\therefore \quad x = +\frac{5}{2} + \frac{\sqrt{13}}{2}$

or $x = +\frac{5}{2} - \frac{\sqrt{13}}{2}$