

Composite Functions

Let $f(x) = x + 3$

$$g(x) = 2x$$

$$h(x) = x^2$$

Find $f(2)$, $g(3)$, $h(4)$

$$\begin{aligned}f(2) &= 2+3 & g(3) &= 2(3) & h(4) &= 4^2 \\f(2) &= 5 & g(3) &= 6 & h(4) &= 16\end{aligned}$$

Find $fg(x)$ $gh(x)$ $hf(x)$

$$fg(x) = f(2x) = 2x + 3$$

$$gh(x) = g(x^2) = 2x^2$$

$$hf(x) = h(x+3) = (x+3)^2$$

Find $fg h(x)$ $h f g(x)$ $g h f(x)$

$$fg h(x) \quad fg(x^2) = f(2x^2) = 2x^2 + 3$$

$$h f g(x) = h f(2x) = h(2x+3) = (2x+3)^2$$

$$g h f(x) = g h(x+3) = g((x+3)^2) = 2(x+3)^2$$

Iteration

$$x^3 - 4x + 1 = 0$$

$$x^3 = 4x - 1$$

$$x = \sqrt[3]{4x - 1}$$

$$x_{n+1} = \sqrt[3]{4x_n - 1}$$

$$x_1 = 2$$

$$x_2 = \sqrt[3]{4x_1 - 1} = 1.913$$

$$x_3 = \sqrt[3]{4x_2 - 1} = 1.881$$

$$x_4 = \sqrt[3]{4x_3 - 1} = 1.868$$

$$x_5 = \sqrt[3]{4x_4 - 1} = 1.864$$

$$x_6 = \sqrt[3]{4x_5 - 1} = 1.862$$

$x = 1.86$ to 2 d.p.

Ex 2

$$x^3 - x^2 - 6 = 0$$

$$x^3 - 6 = x^2$$

$$\sqrt{x^3 - 6} = x$$

$$x_{n+1} = \sqrt{x_n^3 - 6}$$

$$x_1 = 2$$

$$x_2 = \sqrt{2^3 - 6} = 1.414$$

$$x_3 = \sqrt{1.414^3 - 6} \quad \text{no solution!!}$$

Try another rearrangement

$$x^3 - x^2 - 6 = 0$$

$$x^3 = x^2 + 6$$

$$x = \sqrt[3]{x^2 + 6}$$

$$x_{n+1} = \sqrt[3]{x_n^2 + 6}$$

$$x_1 = 2$$

$$x_2 = \sqrt[3]{2^2 + 6} = 2.154$$

$$x_3 = \sqrt[3]{2.154^2 + 6} = 2.199$$

$$x_4 = \sqrt[3]{2.199^2 + 6} = 2.213$$

$$x_5 = \sqrt[3]{2.213^2 + 6} = 2.217$$

$$x_6 = \sqrt[3]{2.217^2 + 6} = 2.218$$

$$x = 2.22 \quad \text{to } 2 \text{ d.p}$$

Ex 3

$$x^3 + 5x - 4 = 0$$

$$x^3 + 5x = 4$$

$$x(x^2 + 5) = 4$$

$$x = \frac{4}{x^2 + 5}$$

$$x_{n+1} = \frac{4}{(x_n^2 + 5)}$$

$$x_0 = 0$$

$$x_1 = \frac{4}{(0^2 + 5)} = 0.8$$

$$x_2 = \frac{4}{(0.8^2 + 5)} = 0.7092$$

$$x_3 = \frac{4}{(0.7092^2 + 5)} = 0.7269$$

$$x_4 = \frac{4}{(0.7269^2 + 5)} = 0.7235$$

Showing roots between given values

Show that the equation

$x^3 + 5x - 4 = 0$ has a solution between
 $x=0$ and $x=1$

$$x=0 \quad 0^3 + 5(0) - 4 = -4$$

$$x=1 \quad 1^3 + 5(1) - 4 = +2$$

Change in sign between 0 and 1,

Since function is continuous there is a root
between 0 and 1