Geometric Progressions

1. For the G.P. 7, 21, 63, 189, ...
Find the 10th term and the sum of the first 8 terms, S_8

$a = 7 \qquad r = 3$

10th term $= ar^9 = 7 \times 3^9 = 137,781$

$S_n = \dfrac{a(r^n - 1)}{r - 1} \qquad S_8 = \dfrac{7(3^8 - 1)}{3 - 1} = 22,960$

2. For the GP 6, 3, 1·5, 0·75, ...
Find the sum to infinity, S_∞

$a = 6 \qquad r = \frac{1}{2}$

$S_\infty = \dfrac{a}{1-r} = \dfrac{6}{1-\frac{1}{2}} = 12$

3. The 4th term of a GP is 96 and the 7th term is 6144. Find the sum of the first 8 terms, S_8.

$4^{th} \quad ar^3 = 96 \qquad (1)$

$7^{th} \quad ar^6 = 6144$

$\dfrac{ar^6}{ar^3} = \dfrac{6144}{96}$

$$r^3 = 64$$

$$r = \sqrt[3]{64} = 4$$

Sub in ①

$$a \times 4^3 = 96$$

$$a = \frac{96}{64} = 1.5$$

$a = 1.5$, $r = 4$

$$S_n = \frac{a(r^n - 1)}{r - 1} \qquad S_8 = \frac{1.5(4^8 - 1)}{4 - 1}$$

$$S_8 = 32767.5$$

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4. The 1st term of a GP is 57 and the sum to infinity is 142.5. Find the common ratio.

$a = 57$ $\qquad S_\infty = \frac{a}{1-r}$

$$\frac{57}{1-r} = 142.5$$

$$57 = 142.5(1-r)$$

$$57 = 142.5 - 142.5r$$

$$142.5r = 142.5 - 57$$

$$r = \frac{142.5 - 57}{142.5}$$

$$r = 0.6$$

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5. The first term of a GP is 5 and the common ratio is 3. Which is the first term to exceed 250,000?

n^{th} term

$$ar^{n-1} > 250000$$

$$5 \times 3^{n-1} > 250000$$

$$3^{n-1} > 50000$$

$$\ln 3^{n-1} > \ln 50000$$

$$(n-1)\ln 3 > \ln 50000$$

$$n-1 > \frac{\ln 50000}{\ln 3}$$

$$n > \frac{\ln 50000}{\ln 3} + 1$$

$$n > 10.848$$

$$\underline{n = 11}$$

6. In the G.P 2, 6, 18, 54, ... how many terms are required for the sum to exceed 5,000,000 ?

$a = 2 \qquad r = 3$

$$S_n = \frac{a(r^n - 1)}{r - 1} > 5{,}000{,}000$$

$$\frac{2(3^n-1)}{3-1} > 5{,}000{,}000$$

$$3^n > 5{,}000{,}001$$

$$\ln 3^n > \ln 5{,}000{,}001$$

$$n \ln 3 > \ln 5000001$$

$$n > \frac{\ln 5000001}{\ln 3}$$

$$n > 14.04$$

$$\underline{n = 15}$$