Name: Solutions

GCSE (1 – 9)

Compound and Inverse Functions

Instructions

• Use **black** ink or ball-point pen.
• Answer all questions.
• Answer the questions in the spaces provided
– *there may be more space than you need.*
• Diagrams are **NOT** accurately drawn, unless otherwise indicated.
• You must **show all your working out.**

Information

• The marks for each question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

• Read each question carefully before you start to answer it.
• Keep an eye on the time.
• Try to answer every question.
• Check your answers if you have time at the end

1. Given that $f(x)=x-4$ find:

a) $f(5)$ $= 5-4 = 1$

1 ..... (1)

b) $f(3)$ $= 3-4 = -1$

-1 ..... (1)

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2. Given that $g(x)=2x^2-10$ find:

a) $g(2)$ $= 2(2)^2 - 10$
$= 8 - 10$
$= -2$

-2 ..... (1)

b) $g(-2)$ $= 2(-2)^2 - 10$
$= 8 - 10$
$= -2$

-2 ..... (1)

c) *Solve*: $g(x)=8$

$2x^2 - 10 = 8$
$2x^2 = 18$
$x^2 = 9$
$x = \pm 3$

$x = \pm 3$ ..... (3)

3. Given that $f(x)=3x-5$ find:

a) $f(3)$

$= 3(3)-5$
$= 9-5$
$= 4$

.....4..... (1)

b) $f(-2)$

$= 3(-2)-5$
$= -6-5$
$= -11$

.....-11..... (1)

c) *Solve*: $f(x)=1$

$3x-5=1$
$3x=1+5$
$3x=6$
$x=2$

.....$x=2$..... (2)

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4. Given that $f(x)=x^2-3$ find:

a) $f(10)$

$= 10^2-3$
$= 100-3$
$= 97$

.....97..... (1)

b) $f(-1)$

$= (-1)^2-3$
$= 1-3$
$= -2$

.....-2..... (1)

c) *Find*: $f^{-1}(x)$

$y = x^2-3$

Swap $x = y^2-3$

$x+3 = y^2$

$\sqrt{x+3} = y$

$f^{-1}(x) = \sqrt{x+3}$

..... (2)

5. Given that $f(x)=2x-4$ and $g(x)=3x+5$

a) Find: $gf(3)$ $= g(2(3)-4) = g(2) = 3(2)+5$

$= 11$

Alternative find $gf(x)$

$= g(2x-4) = 3(2x-4)+5$

$= 6x-12+5$

$gf(x) = 6x-7$

$gf(3) = 6(3)-7$

$= 18-7 = 11$

11 ..... (2)

b) Work out an expression for: $f^{-1}(x)$

Let $y = 2x-4$

Swap $x = 2y-4$

$x+4 = 2y$

$\frac{x+4}{2} = y$

$f^{-1}(x) = \frac{x+4}{2}$ ..... (2)

c) Solve: $f(x)=g(x)$

$2x-4 = 3x+5$

$-4-5 = 3x-2x$

$-9 = x$

$x = -9$ ..... (2)

6. Given that $f(x)=3x+1$ and $g(x)=x^2$

a) Write down an expression for: $fg(x)$

$fg(x) = f(x^2) = 3x^2+1$

$fg(x) = 3x^2+1$ ..... (2)

b) Work out an expression for: $gf(x)$

$gf(x) = g(3x+1) = (3x+1)^2$

$gf(x) = (3x+1)^2$ ..... (2)

c) Solve: $fg(x)=gf(x)$

$3x^2+1 = (3x+1)^2$

$3x^2+1 = 9x^2+6x+1$

$0 = 6x^2+6x$

$0 = 6x(x+1)$

$\Rightarrow x=0$ or $x=-1$ ..... (3)

7. Given that $f(x) = x^2 - 17$ and $g(x) = x + 3$

a) Work out an expression for: $g^{-1}(x)$

Let $y = x + 3$

Swap $x = y + 3$

$x - 3 = y$

$g^{-1}(x) = x - 3$

..... (2)

b) Work out an expression for: $f^{-1}(x)$

Let $y = x^2 - 17$

Swap $x = y^2 - 17$

$x + 17 = y^2$

$\sqrt{x+17} = y$

$f^{-1}(x) = \sqrt{x+17}$

..... (2)

c) Solve: $f^{-1}(x) = g^{-1}(x)$

$\sqrt{x+17} = x - 3$

$x + 17 = x^2 - 6x + 9$

$0 = x^2 - 7x - 8$

$0 = (x - 8)(x + 1)$

$x = 8$ or $x = -1$

check it not correct

$x = 8$

..... (4)

8. A function f is defined such that

$$f(x)=x^2-1$$

a) Find and expression for : $f(x-2)$

$f(x-2) = (x-2)^2 - 1$

..... (2)

b) Hence solve: $f(x-2)=0$

$(x-2)^2 - 1 = 0$

$(x-2)^2 = 1$

$x - 2 = \pm 1$

$x = \pm 1 + 2$

$x = 3$ or $x = 1$

..... (2)

9. A function f is defined such that

$$f(x) = 4x - 1$$

a) Find: $f^{-1}(x)$

Let $y = 4x - 1$

Swap $x = 4y - 1$

$x + 1 = 4y$

$\frac{x+1}{4} = y$

$f^{-1}(x) = \frac{x+1}{4}$ ..... (2)

The function g is such that

$$g(x) = kx^2 \text{ where } k \text{ is a constant}$$

Given that $fg(2) = 12$

b) Work out the value of k

$fg(2) = f(k \times 2^2) = f(4k)$

$= 4(4k) - 1$

$= 16k - 1$

If $fg(2) = 12$

then $16k - 1 = 12$

$16k = 13$

$k = \frac{13}{16}$

$k = \frac{13}{16}$ ..... (2)

19 For all values of x

$$f(x) = (x + 1)^2 \quad \text{and} \quad g(x) = 2(x - 1)$$

(a) Show that $gf(x) = 2x(x + 2)$

$$gf(x) = g((x+1)^2) = 2[(x+1)^2 - 1]$$
$$= 2[x^2 + 2x + 1 - 1]$$
$$= 2[x^2 + 2x]$$
$$= 2x(x+2)$$

(b) Find $g^{-1}(7)$

Let $y = 2(x-1)$

Swap $x = 2(y-1)$

$$\frac{x}{2} = y - 1$$
$$\frac{x}{2} + 1 = y$$
$$g^{-1}(x) = \frac{x}{2} + 1$$
$$g^{-1}(7) = \frac{7}{2} + 1 = \frac{9}{2}$$