Ex1 We spin a coin we believe is biased in favour of heads. It is spun 20 times. Carry out a Hypothesis test at the 5% significance level to conclude whether it is biased. Suppose 14 heads are obtained

Let X be number of heads

$$X \sim B(\overset{n}{20}, \overset{p}{\tfrac{1}{2}})$$

$H_0 : p = 0.5$

$H_1 : p > 0.5$

where p is prob of obtaining a head on any random spin

$$P(X \geq 14)$$

$$= 1 - P(X \leq 13)$$

$$= 1 - 0.9423$$

$$= 0.0577 \quad > 5\%$$

Accept H_0 There is not sufficient evidence to support the view prob(Head) > 0.5

Conclude the coin is fair

Ex2 On average 45% of driving tests result in a pass. To decide whether an examiner is too harsh or too lenient a test is set up at the 10% sig level on a sample of 20 of the examiner tests. Find the critical region

Let X be number of passes

then $X \sim B(20, 0.45)$

$H_0 : p = 0.45$

$H_1 : p <> 0.45$

10% sig level

so 5% each end

$P(X \leq 4) = 0.0189 < 5\%$

$P(X \leq 5) = 0.0553 > 5\%$

C.R. at bottom end $= \{0, 1, 2, 3, 4\}$

$P(X \geq 12) = 1 - P(X \leq 11)$

$= 1 - 0.8692 = 0.1308 > 5\%$

$P(X \geq 13) = 1 - P(X \leq 12)$

$= 1 - 0.9419 = 0.0581 > 5\%$

$P(X \geq 14) = 1 - P(X \leq 13)$

$= 1 - 0.9785 = 0.0215 < 5\%$

$\therefore$ top end CR $= \{14, 15, 16, 17, 18, 19, 20\}$

Critical Region $X \leq 4 \cup X \geq 14$

---

7 A type of shampoo is known to relieve the symptoms of 75% of dogs who suffer from a particular minor allergy.

(i) 12 dogs who suffer from this allergy are selected at random. Find the probability that the number of these dogs who have their symptoms relieved is

(*A*) exactly 9, **[3]**

(*B*) at least 9. **[2]**

A new type of shampoo has been developed to treat the allergy. A hypothesis test is to be carried out to determine whether it relieves the symptoms of a higher proportion of dogs who suffer from the allergy.

(ii) Write down suitable null and alternative hypotheses for the test. Give a reason for your choice of alternative hypothesis. **[4]**

A random sample of n dogs who suffer from the allergy is selected.

(iii) (*A*) Given that $n = 18$ and the symptoms of 16 dogs are relieved, carry out the test at the 10% significance level. **[4]**

(*B*) Given instead that $n = 50$ and the symptoms of 42 dogs are relieved, carry out the test at the 10% significance level. You may use the information that, for $X \sim B(50, 0.75)$,

$P(X = 41) = 0.0721$, $P(X = 42) = 0.0463$, $P(X \leqslant 41) = 0.9084$, $P(X \leqslant 42) = 0.9547$. **[4]**

i) A) $X \sim B(\overset{n}{12}, \overset{p}{0.75})$

$P(X=9) = {}^{12}C_9\, 0.75^9 \times 0.25^3$

$= 0.2581$

B) $P(X \geq 9) = 1 - P(X \leq 8)$

$= 1 - 0.3512$

$= \underline{0.6488}$

ii) $H_0 : p = 0.75$

$H_1 : p > 0.75$

p = prob a randomly chosen dog has symptoms relieved by new shampoo

H_1 chosen as $p > 0.75$ as improvement tested for.

---

iii) $X \sim B(\overset{n}{18}, \overset{p}{0.75})$

10% sig level

$P(X \geq 16)$

$= 1 - P(X \leq 15)$

$= 1 - 0.8647 = 0.1353 > 10\%$

Accept H_0. There is not sufficient evidence to support the view the shampoo relieves a greater proportion of dogs. Accept still >75% of dogs relieved.
