COMPLETING THE SQUARE

TRANSCRIPT

For quadratic functions, completing the square is about writing:

$$y = ax^2 + bx + c$$

in the form:

$$y = a(x+p)^2 + q$$

Consider

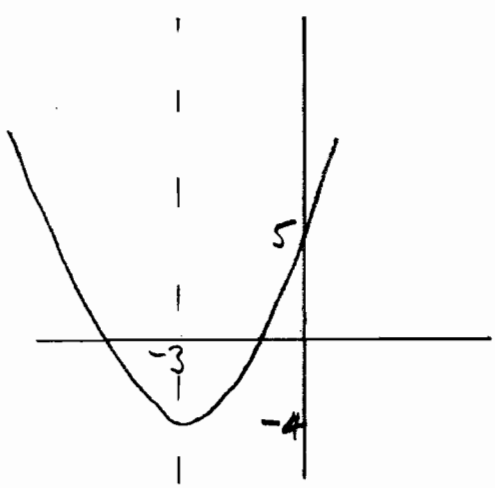
$$(x+p)^2 = (x+p)(x+p) = x^2 + px + px + p^2 = x^2 + 2px + p^2$$

This tells us that the number in the bracket should be half the amount of x we have:

Example

$$y = x^2 + 6x + 5$$
$$y = (x+3)^2 + 5 - 9$$
$$y = (x+3)^2 - 4$$

The 9 is subtracted because $(x+3)(x+3)$ introduces a term 3^2 which was not in the original function



Line of symmetry: $x = -3$

Minimum point: $(-3, -4)$

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Consider

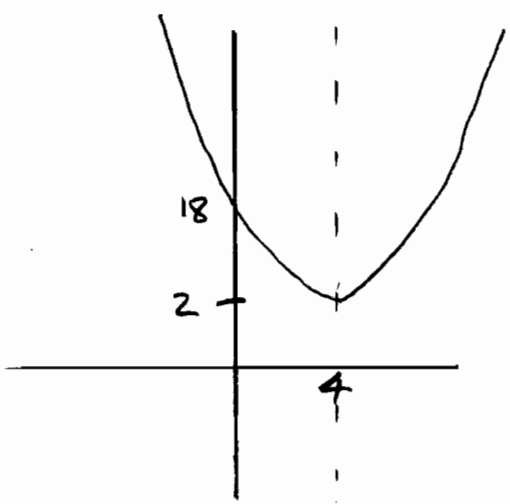
$$(x-p)^2 = (x-p)(x-p) = x^2 - px - px + p^2 = x^2 - 2px + p^2$$

Again the number in the bracket is half the amount of x we have, and again an extra term is introduced which will need to be subtracted.

Example

$$y = x^2 - 8x + 18$$
$$y = (x-4)^2 + 18 - 16$$
$$y = (x-4)^2 + 2$$

The 16 is subtracted because $(x-4)(x-4)$ introduces a term $(-4)^2$ which was not in the original function.



Line of symmetry: $x = 4$

Minimum point: $(4, 2)$

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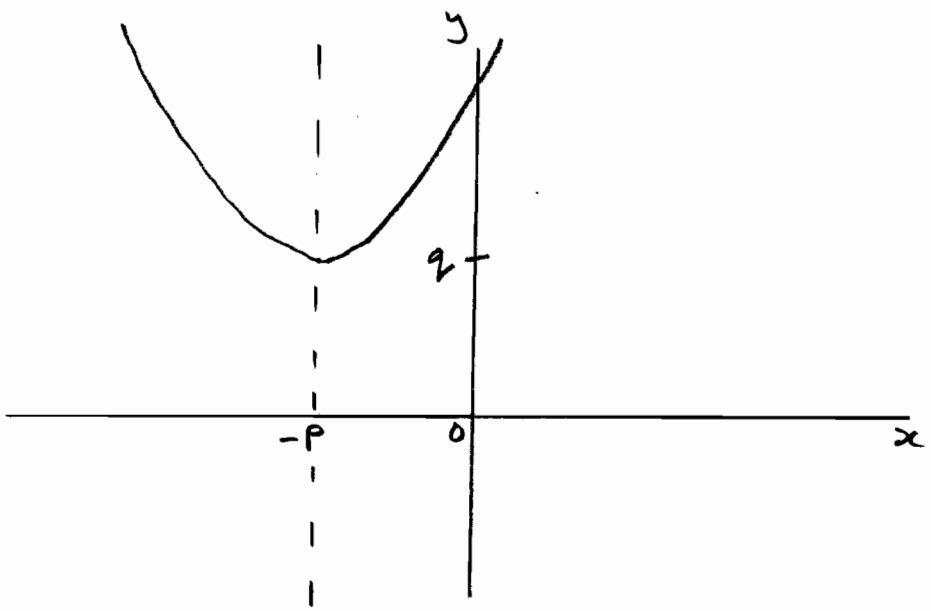
Summary:

If $y = (x+p)^2 + q$

Then there is:

A line of symmetry at $x = -p$

A minimum point at $(-p, q)$



Write the following functions in the form: $y = a(x+p)^2 + q$

Identify the turning point at $(-p, q)$

and the line of symmetry at $x = -p$

Sketch the graphs

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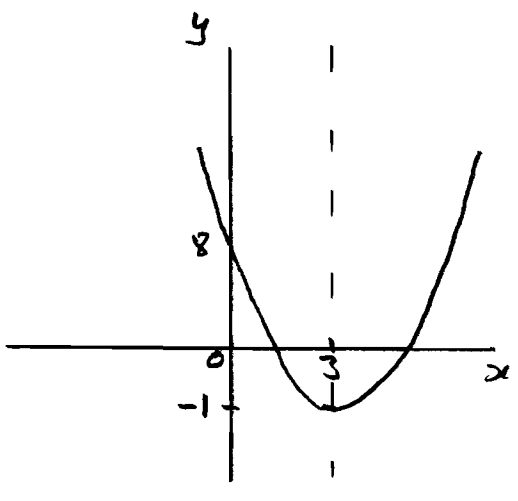
Example 1

$$y = x^2 - 6x + 8$$
$$y = (x-3)^2 + 8 - 9$$
$$y = (x-3)^2 - 1$$

Line of symmetry $x = 3$

Minimum point $(3, -1)$

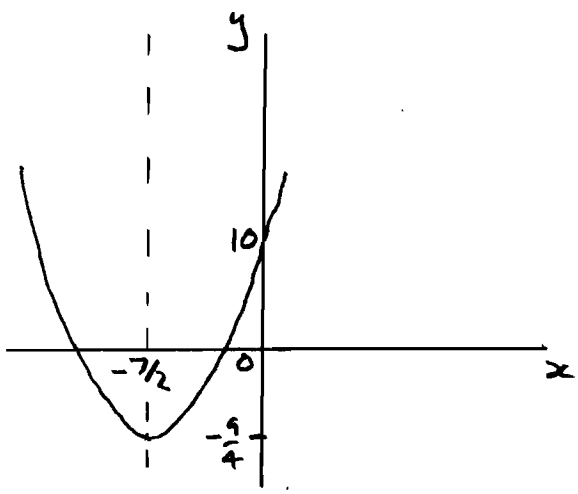


Example 2

$$y = x^2 + 7x + 10$$
$$y = \left(x + \tfrac{7}{2}\right)^2 + 10 - \tfrac{49}{4}$$
$$y = \left(x + \tfrac{7}{2}\right)^2 - \tfrac{9}{4}$$

Line of symmetry $x = -\frac{7}{2}$

Minimum point $\left(-\frac{7}{2}, -\frac{9}{4}\right)$



COMPLETING THE SQUARE

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Example 3

$$y = 2x^2 - 10x + 13$$

$$y = 2\left[x^2 - 5x + \tfrac{13}{2}\right]$$

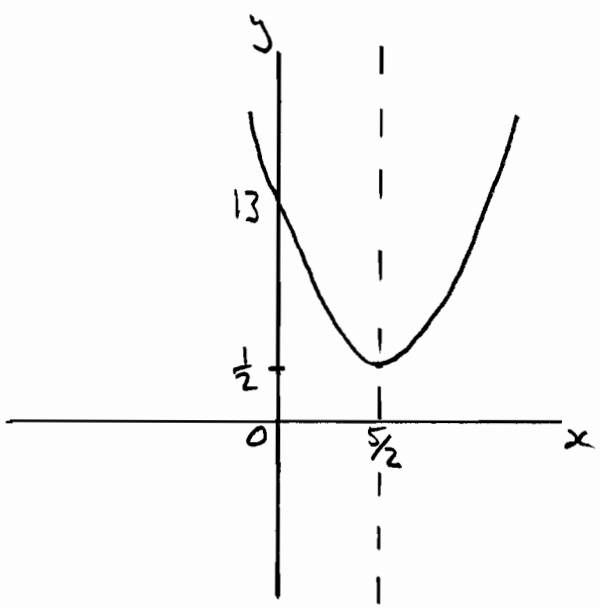
$$y = 2\left[(x - \tfrac{5}{2})^2 + \tfrac{13}{2} - \tfrac{25}{4}\right]$$

$$y = 2\left[(x - \tfrac{5}{2})^2 + \tfrac{1}{4}\right]$$

$$y = 2(x - \tfrac{5}{2})^2 + \tfrac{1}{2}$$

Line of symmetry $x = \frac{5}{2}$

Minimum point $(\frac{5}{2}, \frac{1}{2})$



Example 4

$$y = -3x^2 + 12x + 7$$

$$y = -3\left[x^2 - 4x - \tfrac{7}{3}\right]$$

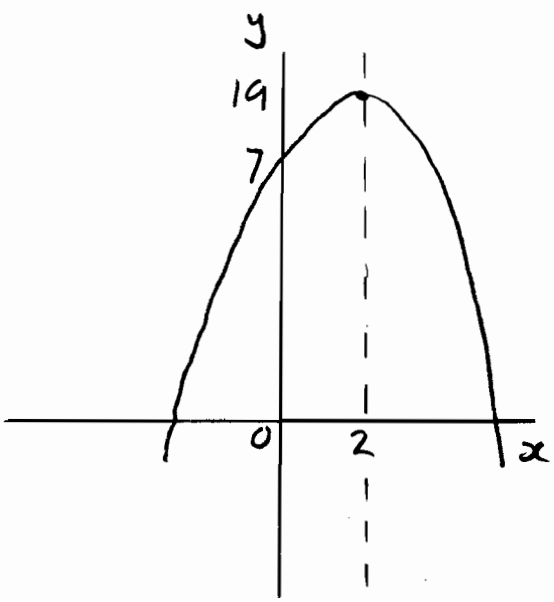
$$y = -3\left[(x-2)^2 - \tfrac{7}{3} - 4\right]$$

$$y = -3\left[(x-2)^2 - \tfrac{19}{3}\right]$$

$$y = -3(x-2)^2 + 19$$

Line of symmetry $x = 2$

Maximum point $(2, 19)$



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Solving equations :

Example 5

$$x^2 - 2x - 8 = 0$$
$$(x-1)^2 - 8 - 1 = 0$$
$$(x-1)^2 - 9 = 0$$
$$(x-1)^2 = 9$$
$$x - 1 = \pm 3$$
$$x = +1 \pm 3$$
$$x = +4 \text{ or } x = -2$$

Example 6

$$x^2 - 3x - 10 = 0$$
$$\left(x - \tfrac{3}{2}\right)^2 - 10 - \tfrac{9}{4} = 0$$
$$\left(x - \tfrac{3}{2}\right)^2 - \tfrac{49}{4} = 0$$
$$\left(x - \tfrac{3}{2}\right)^2 = \tfrac{49}{4}$$
$$x - \tfrac{3}{2} = \pm \tfrac{7}{2}$$
$$x = +\tfrac{3}{2} \pm \tfrac{7}{2}$$

Either $x = \frac{10}{2} = 5$

or $x = -\frac{4}{2} = -2$

$x = 5$ or $x = -2$

Example 7

$$x^2 + 6x + 2 = 0$$
$$(x+3)^2 + 2 - 9 = 0$$
$$(x+3)^2 - 7 = 0$$
$$(x+3)^2 = 7$$
$$x + 3 = \pm\sqrt{7}$$
$$x = -3 \pm \sqrt{7}$$

$x = -3 + \sqrt{7}$

or $x = -3 - \sqrt{7}$