

# Completing the Square

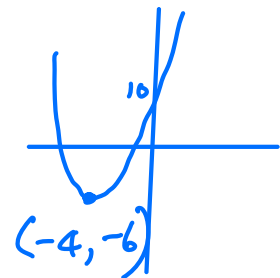
$$\begin{aligned}\text{Consider } & (x+a)^2 \\ &= (x+a)(x+a) \\ &= x^2 + ax + ax + a^2 \\ &= x^2 + 2ax + a^2\end{aligned}$$

Completing the square makes use of the relationship above

Ex 1 Complete the square for

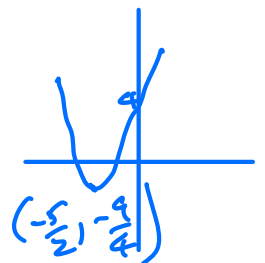
$$\begin{aligned}x^2 + 8x + 10 \\ &= (x+4)^2 + 10 - 16 \\ &= (x+4)^2 - 6\end{aligned}$$

$$\begin{aligned}(x+a)(x+a) \\ &= x^2 + 8x + 16\end{aligned}$$



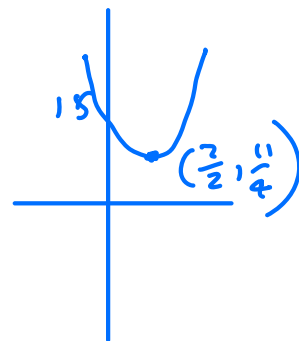
Ex 2

$$\begin{aligned}x^2 + 5x + 4 \\ &= \left(x + \frac{5}{2}\right)^2 + 4 - \frac{25}{4} \\ &= \left(x + \frac{5}{2}\right)^2 - \frac{9}{4}\end{aligned}$$



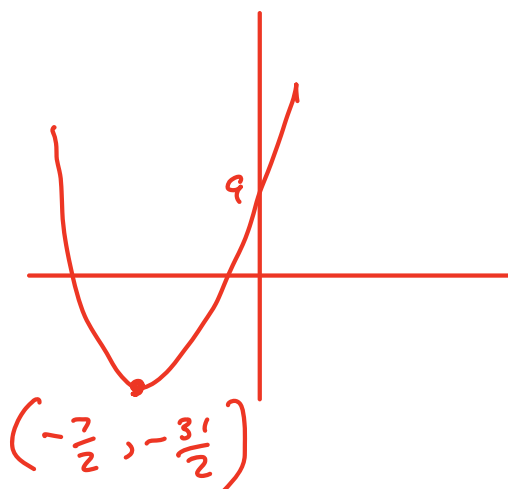
Ex 3

$$\begin{aligned} & x^2 - 7x + 15 \\ &= \left(x - \frac{7}{2}\right)^2 + 15 - \frac{49}{4} \\ &= \left(x - \frac{7}{2}\right)^2 + \frac{60}{4} - \frac{49}{4} \\ &= \left(x - \frac{7}{2}\right)^2 + \frac{11}{4} \end{aligned}$$



Ex 4

$$\begin{aligned} & 2x^2 + 14x + 9 \\ & 2 \left[ x^2 + 7x + \frac{9}{2} \right] \\ & 2 \left[ \left(x + \frac{7}{2}\right)^2 + \frac{9}{2} - \frac{49}{4} \right] \\ & 2 \left( x + \frac{7}{2} \right)^2 + 9 - \frac{49}{2} \\ &= 2 \left( x + \frac{7}{2} \right)^2 - \frac{31}{2} \end{aligned}$$



Ex 5

$$\begin{aligned} & 3x^2 + 12x + 5 \\ = & 3 \left[ x^2 + 4x + \frac{5}{3} \right] \\ = & 3 \left[ (x+2)^2 + \frac{5}{3} - 4 \right] \\ = & 3(x+2)^2 + 5 - 12 \\ = & 3(x+2)^2 - 7 \end{aligned}$$

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Ex 6

$$y = -4x^2 + 20x - 15$$

$$y = -4 \left[ x^2 - 5x + \frac{15}{4} \right]$$

$$y = -4 \left[ \left( x - \frac{5}{2} \right)^2 + \frac{15}{4} - \frac{25}{4} \right]$$

$$y = -4 \left[ \left( x - \frac{5}{2} \right)^2 - \frac{10}{4} \right]$$

$$y = -4 \left( x - \frac{5}{2} \right)^2 + 10$$

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