

# Intro to Vectors

A vector quantity has both magnitude and direction

Examples velocity, acceleration, weight, displacement

A scalar quantity has only magnitude

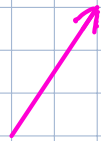
Examples Mass, Length

## Representing Vectors

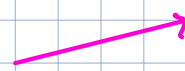
In text books and exam papers vectors are shown in bold print. When handwriting they should always be underlined.

Examples

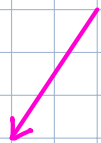
$$\underline{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$



$$\underline{b} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$



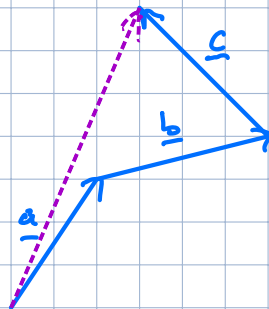
$$-\underline{a} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$$



$$\underline{c} = \begin{pmatrix} -3 \\ 3 \end{pmatrix}$$



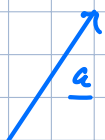
$$\underline{a} + \underline{b} + \underline{c}$$

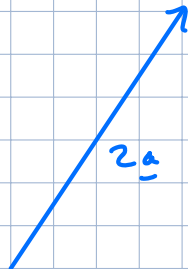


$$\text{Resultant } \underline{a} + \underline{b} + \underline{c} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ 1 \end{pmatrix} + \begin{pmatrix} -3 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$$

This is the nose to tail rule for adding vectors

# Scalar Multiples of a Vector

$$\underline{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$


$$2\underline{a} = 2 \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$$


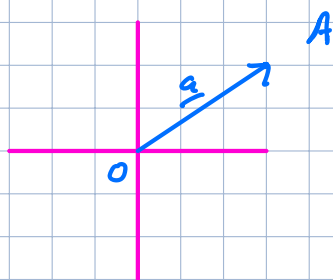
Notice  $2\underline{a}$  has twice magnitude and is in the same direction as  $\underline{a}$

The magnitude of  $\underline{a}$  written as  $|\underline{a}| = \sqrt{2^2 + 3^2} = \sqrt{13}$

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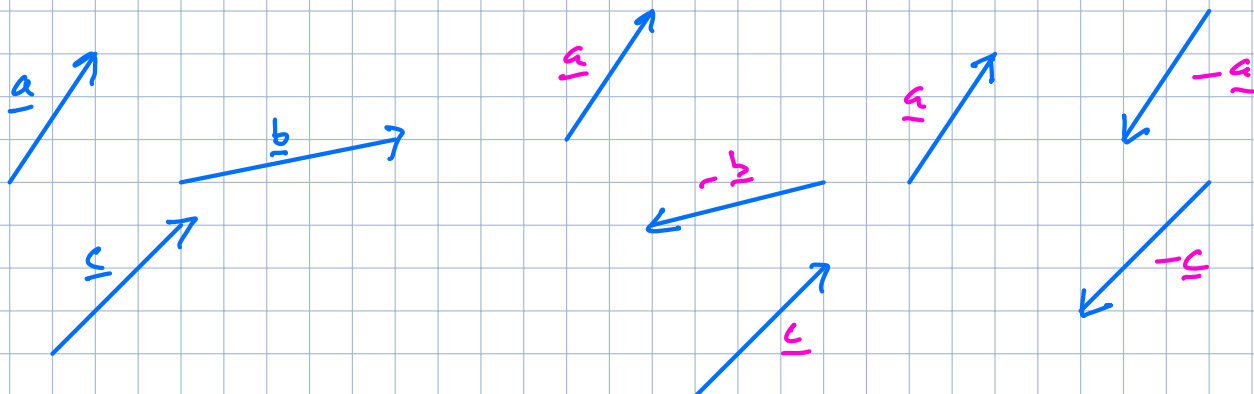
In general vectors do not have position although there are such things as position vectors which relate points to the origin

eg

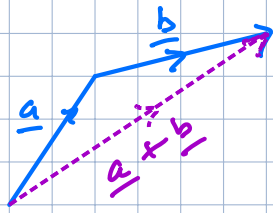


$\underline{a} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$  is the position vector of  $A(3, 2)$

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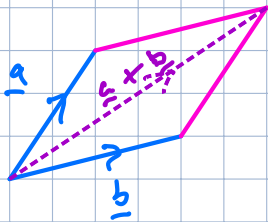


## Triangle Rule For Adding Vectors



If  $\underline{a}$  and  $\underline{b}$  are nose to tail  
 $\underline{a} + \underline{b}$  is the third side  
of the triangle

## Parallelogram Rule for Adding Vectors



## Working With Column Vectors

$$\text{Let } \underline{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad \underline{b} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \quad \underline{c} = \begin{pmatrix} -3 \\ 3 \end{pmatrix}$$

Find

$$\begin{aligned} 1) \quad 2\underline{a} + 3\underline{b} &= 2 \begin{pmatrix} 2 \\ 3 \end{pmatrix} + 3 \begin{pmatrix} 4 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} 4 \\ 6 \end{pmatrix} + \begin{pmatrix} 12 \\ 3 \end{pmatrix} = \begin{pmatrix} 16 \\ 9 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} 2) \quad 4\underline{a} - 3\underline{c} &= 4 \begin{pmatrix} 2 \\ 3 \end{pmatrix} - 3 \begin{pmatrix} -3 \\ 3 \end{pmatrix} \\ &= \begin{pmatrix} 8 \\ 12 \end{pmatrix} + \begin{pmatrix} 9 \\ -9 \end{pmatrix} \end{aligned}$$

$$= \begin{pmatrix} 17 \\ 3 \end{pmatrix}$$

Exercise

$$\underline{a} = \begin{pmatrix} 7 \\ 2 \end{pmatrix} \quad \underline{b} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \quad \underline{c} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

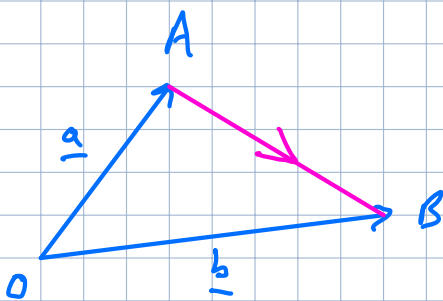
Find

$$1) \quad -2\underline{a} = -2 \begin{pmatrix} 7 \\ 2 \end{pmatrix} = \begin{pmatrix} -14 \\ -4 \end{pmatrix}$$

$$2) \quad \underline{b} + 2\underline{c} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + 2 \begin{pmatrix} -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + \begin{pmatrix} -4 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

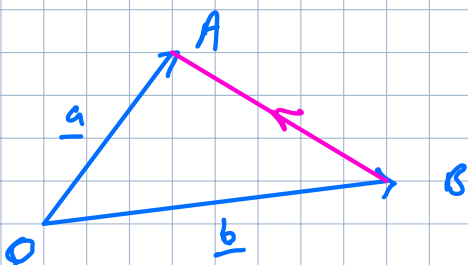
$$3) \quad \underline{a} + 2\underline{b} + 3\underline{c}$$
$$= \begin{pmatrix} 7 \\ 2 \end{pmatrix} + 2 \begin{pmatrix} 3 \\ 0 \end{pmatrix} + 3 \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$
$$= \begin{pmatrix} 7 \\ 2 \end{pmatrix} + \begin{pmatrix} 6 \\ 0 \end{pmatrix} + \begin{pmatrix} -6 \\ 3 \end{pmatrix} = \begin{pmatrix} 7 \\ 5 \end{pmatrix}$$

## Important Results



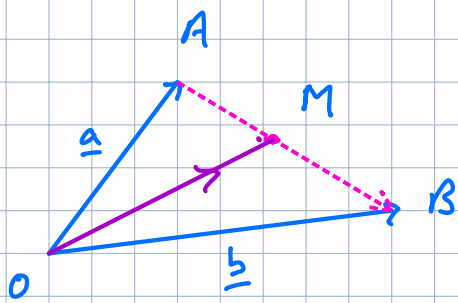
Find  $\vec{AB}$

$$\vec{AB} = -\underline{a} + \underline{b}$$



Find  $\vec{BA}$

$$\vec{BA} = -\underline{b} + \underline{a}$$



Let midpoint of AB be M  
Find  $\vec{OM}$

$$\begin{aligned}\vec{OM} &= \vec{OA} + \vec{AM} \\ &= \vec{OA} + \frac{1}{2} \vec{AB} \\ &= \underline{a} + \frac{1}{2} (-\underline{a} + \underline{b}) \\ &= \underline{a} - \frac{1}{2} \underline{a} + \frac{1}{2} \underline{b} \\ &= \frac{1}{2} \underline{a} + \frac{1}{2} \underline{b} \\ &\text{or } \frac{1}{2} (\underline{a} + \underline{b})\end{aligned}$$