

Binomial Expansion

$$(a+b) = a+b$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a+b)^3 = (a+b)^2(a+b)$$

$$= (a^2 + 2ab + b^2)(a+b)$$

$$\begin{aligned} &= \frac{a^3 + 2a^2b + ab^2}{a^3 + 3a^2b + 3ab^2 + b^3} \\ &\quad + a^2b + 2ab^2 + b^3 \end{aligned}$$

$$(a+b)^4 = (a^3 + 3a^2b + 3ab^2 + b^3)(a+b)$$

$$\begin{aligned} &\quad a^4 + 3a^3b + 3a^2b^2 + ab^3 \\ &\quad + a^3b + 3a^2b^2 + 3ab^3 + b^4 \\ \hline &\quad a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4 \end{aligned}$$

$$(a+b)(a+b)(a+b)(a+b)$$

Pascal's Triangle

			1						
			1	2	1				
			1	3	3	1			
			1	4	6	4	1		
			1	5	10	10	5	1	
			1	6	15	20	15	6	1

$$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

$$(a+b)^6 = a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$$

The number of ways of choosing r from n when order does not matter is given by

$$nCr = {}^nC_r \quad \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

(factorial notation $5! = 5 \times 4 \times 3 \times 2 \times 1$)

Aside

How many ways can 4 objects be arranged?

$$\boxed{4 \ 3 \ 2 \ 1} = 4! = 24$$

ABCD	BACD	CABD	DABC
ABDC	BADC	CADB	DACB
ACBD	BCAD	CBAD	DBAC
ACDB	BCDA	CBDA	DBCA
ADBC	BADC	CDAB	DCAB
ADCB	BDC A	CDBA	DCBA

Choose 3 from 4 when order matters

$$\boxed{4 \downarrow 3 \downarrow 2} = 24$$

Choose 2 from 4 when order matters

$$4P_2 \quad \boxed{4 \downarrow 3} = 12$$

Choose r from n when order matters

$$\boxed{n(n-1) \cdots (n-r+1)} = \frac{n!}{(n-r)!}$$

$$\text{Ex } {}^7P_3 = 7 \cdot 6 \cdot 5 = 210$$

$$= \frac{7!}{(7-3)!} = \frac{7!}{4!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1}$$

If order does not matter

ABC	}	all the same
ACB		
BAC		
BCA		
CAB		
CBA		

$$nCr = \frac{n!}{(n-r)!r!}$$

$${}^7C_3 = \frac{7!}{4!3!} = \frac{7 \cdot 6 \cdot 5 \cdot \cancel{4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}} = 35$$

The term in a^r in the expansion
 $(a+b)^n$ is given by $\binom{n}{r} a^r b^{n-r}$

Ex1 Find the term in x^3 in the expansion
 $(2x+3)^7$

$$\begin{aligned}\text{Term in } x^3 &= \binom{7}{3} (2x)^3 (3)^4 \\ &= 35 \times 8x^3 \times 81x^3 \\ &= 22680x^3\end{aligned}$$

The coefficient of x^3 = 22680

- 1 Find the coefficient of x^3 in the expansion of $(2 + 3x)^5$. [4]

- 2 Find the binomial expansion of $(2 - x)^3$. [4]

1) ${}^5C_3 (3x)^3 (2)^2$
 $10 \times 27x^3 \times 4 = 1080x^3$

Coeff = 1080

2) $\begin{array}{cccc} & 1 & 2 & 1 \\ & | & 3 & 3 & | \end{array}$ $2^3 + 3(2)(-x) + 3(2)(-x)^2 + (-x)^3$
 $= 8 - 12x + 6x^2 - x^3$

- 3 Find the binomial expansion of $(2 + x)^4$, writing each term as simply as possible. [4]

- 4 Calculate 6C_3 .

Find the coefficient of x^3 in the expansion of $(1 - 2x)^6$. [4]

$$3) (2+x)^4$$

$$= 2^4 + 4(2)^3x + 6(2)^2x^2 + 4(2)x^3 + x^4$$

$$= 16 + 32x + 24x^2 + 8x^3 + x^4$$

$$4) {}^6C_3 = \frac{6!}{3!3!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1} = 20$$

$$(1-2x)^6$$

Term in $x^3 = {}^6C_3 (1)^3 (-2x)^3$

$$= 20 \times 1 \times (-8x^3)$$

$$= -160x^3$$

Coeff of $x^3 = -160$

Hwk Please complete 5 to 10
from sheet emailed to you.
