Definite Integration

$$\int 3x^2\,dx = x^3 + c$$

$$\int_1^3 3x^2\,dx = \left[x^3 + c\right]_1^3$$

$$= \left[3^3 + c\right] - \left[1^3 + c\right]$$

$$= 27 + \cancel{c} - 1 - \cancel{c}$$

$$= 26$$

Because the constants cancel we omit the constant c in definite integration

$$\int_2^5 4x\,dx = \left[2x^2\right]_2^5$$

$$(2 \times 5^2) - (2 \times 2^2)$$

$$= 50 - 8$$

$$= 42$$

Exercise 13 D

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1d) $$\int_1^3 \frac{3}{x^2}\,dx = \int_1^3 3x^{-2}\,dx$$

$$= \left[\frac{3x^{-1}}{-1}\right]_1^3 = \left[-\frac{3}{x}\right]_1^3$$

$$= \left(-\frac{3}{3}\right) - \left(-\frac{3}{1}\right)$$

$$= -1 + 3$$

$$= 2$$

2d) $$\int_1^8 \left(x^{-\frac{1}{3}} + 2x - 1\right)dx$$

$$= \left[\frac{x^{2/3}}{\frac{2}{3}} + \frac{2x^2}{2} - x\right]_1^8$$

$$= \left[\frac{3}{2}x^{2/3} + x^2 - x\right]_1^8$$

$$= (6 + 64 - 8) - \left(\frac{3}{2} + 1 - 1\right)$$

$$= 62 - \frac{3}{2}$$

$$= 60.5$$

3d) $$\int_1^4 \frac{2 + \sqrt{x}}{x^2}\,dx = \int_1^4 \left(2x^{-2} + x^{-3/2}\right)dx$$

$$= \left[\frac{2x^{-1}}{-1} + \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}}\right]_1^4$$

$$= \left[-\frac{2}{x} - \frac{2}{\sqrt{x}} \right]_1^4$$

$$= \left(-\frac{2}{4} - \frac{2}{\sqrt{4}} \right) - \left(-\frac{2}{1} - \frac{2}{\sqrt{1}} \right)$$

$$= \left(-\frac{1}{2} - 1 \right) - \left(-2 - 2 \right)$$

$$= -\frac{3}{2} + 4$$

$$= 2.5$$

Classwork 1c, 2c, 3c

1c) $$\int_0^4 \sqrt{x}\, dx = \int_0^4 x^{\frac{1}{2}} dx = \left[\frac{x^{3/2}}{3/2} \right]_0^4 = \left[\frac{2}{3} x^{3/2} \right]_0^4$$

$$= \left(\frac{2}{3}(4)^{3/2} \right) - \left(\frac{2}{3} 0^{3/2} \right)$$

$$= \frac{16}{3} - 0 \qquad = \frac{16}{3}$$

2c) $$\int_4^9 \left(\sqrt{x} - \frac{6}{x^2} \right) dx = \int_4^9 \left(x^{\frac{1}{2}} - 6x^{-2} \right) dx$$

$$= \left[\frac{x^{3/2}}{3/2} - \frac{6x^{-1}}{-1} \right]_4^9$$

$$= \left[\frac{2}{3} x^{3/2} + \frac{6}{x} \right]_4^9$$

$$= \left(\frac{2}{3}(9)^{3/2} + \frac{6}{9}\right) - \left(\frac{2}{3}(4)^{3/2} + \frac{6}{4}\right)$$

$$= \left(18 + \frac{2}{3}\right) - \left(\frac{16}{3} + \frac{3}{2}\right)$$

$$= \frac{71}{6}$$

3c) $$\int_0^1 x^2\left(\sqrt{x} + \frac{1}{x}\right)dx = \int_0^1 \left(x^{5/2} + x\right)dx$$

$$= \left[\frac{x^{7/2}}{7/2} + \frac{x^2}{2}\right]_0^1 = \left[\frac{2}{7}x^{7/2} + \frac{x^2}{2}\right]_0^1$$

$$= \left(\frac{2}{7}(1) + \frac{(1)^2}{2}\right) - (0 + 0)$$

$$= \frac{2}{7} + \frac{1}{2}$$

$$= \frac{11}{14}$$

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4) $$\int_1^4 (6\sqrt{x} - A)\,dx = A^2$$

$$= \int_1^4 \left(6x^{\frac{1}{2}} - A\right)dx = A^2$$

$$= \left[\frac{6x^{3/2}}{3/2} - Ax\right]_1^4 = A^2$$

$$= \left[4x^{3/2} - Ax\right]_1^4 = A^2$$

$$(32-4A)-(4-A)=A^2$$

$$28-3A=A^2$$

$$\Rightarrow \quad A^2+3A-28=0 \qquad \therefore \text{ 2 solutions}$$

$$(A-4)(A+7)=0$$

$$\Rightarrow \quad A=4 \quad \text{or} \quad A=-7$$

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8) $V=20+5t \qquad 0 \le t \le 10\text{ s}$

$$s=\int_0^{10}(20+5t)\,dt$$

$$=\left[20t+\frac{5t^2}{2}\right]_0^{10}$$

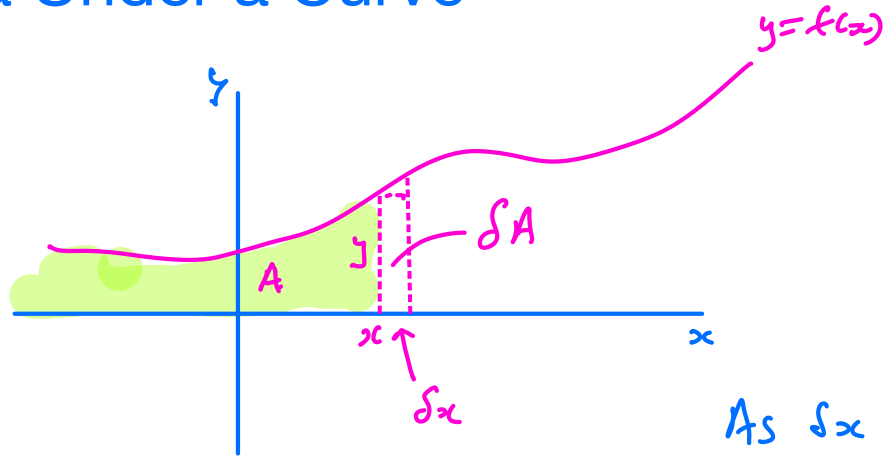
$$=\left(20(10)+\frac{5(10)^2}{2}\right)-(0+0)$$

$$=200+250-0$$

$$=450\text{ m}$$

Classwork Q 5, 6, 7

Area Under a Curve



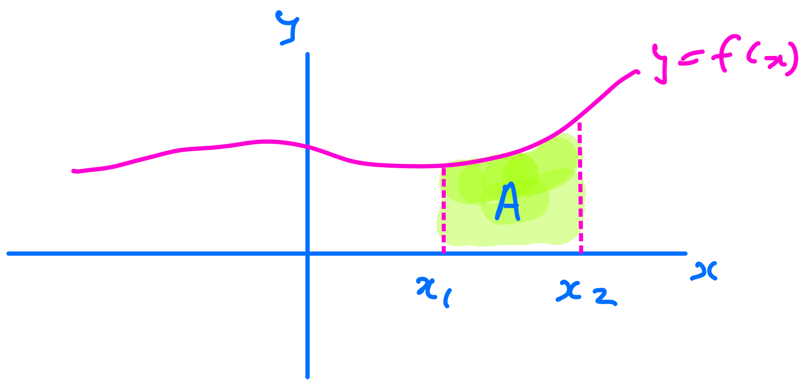
$$\delta A \approx y\,\delta x$$

$$\frac{\delta A}{\delta x} \approx y$$

As $\delta x \to 0$

$$\frac{dA}{dx} = y$$

$$\Rightarrow \quad A = \int y\,dx$$



$$A = \int_{x_1}^{x_2} y\,dx$$