Year 12 Assessment 1 Pure maths

1. (a) Simplify

$$\sqrt{50} - \sqrt{18}$$

giving your answer in the form $a\sqrt{2}$, where a is an integer.

(2)

(b) Hence, or otherwise, simplify

$$\frac{12\sqrt{3}}{\sqrt{50} - \sqrt{18}}$$

giving your answer in the form $b\sqrt{c}$, where b and c are integers and $b \neq 1$

(3)

You must show your method

2.

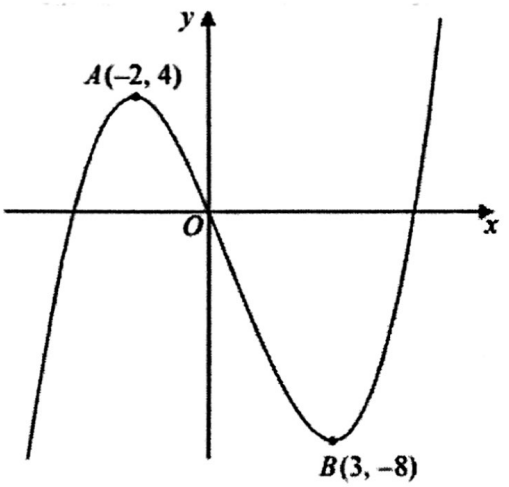

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = f(x)$. The curve has a maximum point A at $(-2, 4)$ and a minimum point B at $(3, -8)$ and passes through the origin O.

On separate diagrams, sketch the curve with equation

(a) $y = 3f(x)$,

(2)

(b) $y = f(x) - 4$

(3)

On each diagram, show clearly the coordinates of the maximum and the minimum points and the coordinates of the point where the curve crosses the y-axis.

3. (a) On separate axes sketch the graphs of

(i) $y = -3x + c$, where c is a positive constant,

(ii) $y = \frac{1}{x} + 5$

On each sketch show the coordinates of any point at which the graph crosses the y-axis and the equation of any horizontal asymptote.

(4)

Given that $y = -3x + c$, where c is a positive constant, meets the curve $y = \frac{1}{x} + 5$ at two distinct points,

(b) show that $(5 - c)^2 > 12$

(3)

(c) Hence find the range of possible values for c.

(4)

4. **(a)** A curve has equation $y = 8 - 4x - 2x^2$.

(i) Find the values of x where the curve crosses the x-axis, giving your answer in the form $m \pm \sqrt{n}$, where m and n are integers.

[2 marks]

(ii) Sketch the curve, giving the value of the y-intercept.

[2 marks]

(b) A line has equation $y = k(x + 4)$, where k is a constant.

(i) Show that the x-coordinates of any points of intersection of the line with the curve $y = 8 - 4x - 2x^2$ satisfy the equation

$$2x^2 + (k + 4)x + 4(k - 2) = 0$$

[1 mark]

(ii) Find the values of k for which the line is a tangent to the curve $y = 8 - 4x - 2x^2$.

[3 marks]

6.

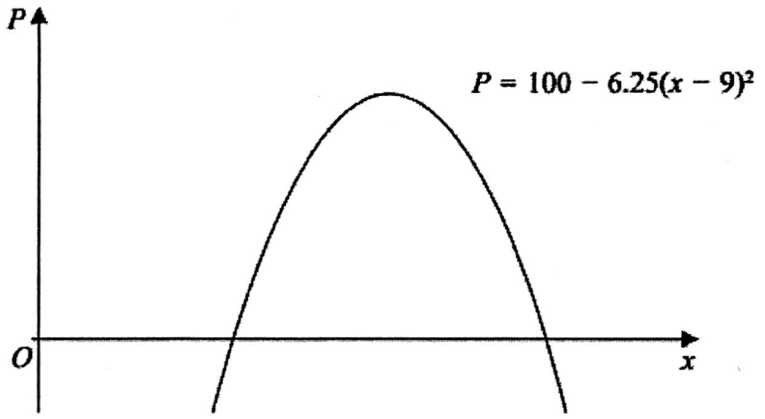

Figure 1

A company makes a particular type of children's toy.

The annual profit made by the company is modelled by the equation

$$P = 100 - 6.25(x - 9)^2$$

where P is the profit measured in thousands of pounds and x is the selling price of the toy in pounds.

A sketch of P against x is shown in Figure 1.

Using the model,

(a) explain why £15 is not a sensible selling price for the toy.

(2)

Given that the company made an annual profit of more than £80000

(b) find, according to the model, the least possible selling price for the toy.

(3)

The company wishes to maximise its annual profit.

State, according to the model,

(c) (i) the maximum possible annual profit,

(ii) the selling price of the toy that maximises the annual profit.

(2)

6.

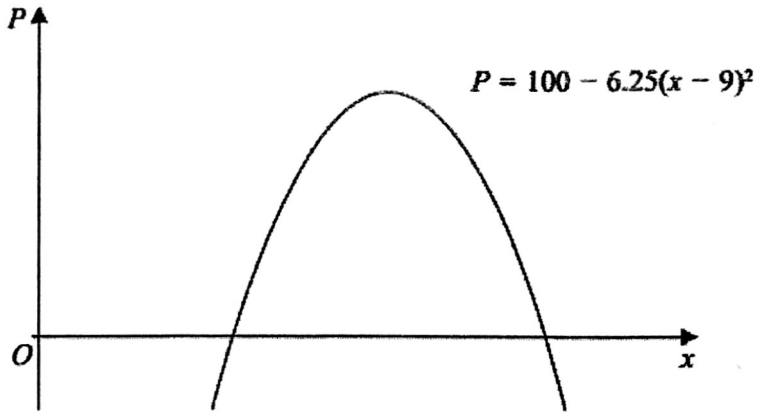

Figure 1

A company makes a particular type of children's toy.

The annual profit made by the company is modelled by the equation

$$P = 100 - 6.25(x - 9)^2$$

where P is the profit measured in thousands of pounds and x is the selling price of the toy in pounds.

A sketch of P against x is shown in Figure 1.

Using the model,

(a) explain why £15 is not a sensible selling price for the toy.

(2)

Given that the company made an annual profit of more than £80 000

(b) find, according to the model, the least possible selling price for the toy.

(3)

The company wishes to maximise its annual profit.

State, according to the model,

(c) (i) the maximum possible annual profit,

(ii) the selling price of the toy that maximises the annual profit.

(2)