Exercise 4A

Q3 $\quad \text{Vol} = \pi \int y^2 \, dx = \pi \int_1^3 (\ln x)^2 \, dx$

$\int (\ln x)^2 \, dx \qquad$ Let $u = (\ln x)^2 \qquad \frac{dv}{dx} = 1$

$\frac{du}{dx} = \frac{2\ln x}{x} \qquad v = x$

$$\int u \frac{dv}{dx} = uv - \int v \frac{du}{dx}$$

$$\int (\ln x)^2 dx = x(\ln x)^2 - \int 2 \ln x \, dx$$

$$= x(\ln x)^2 - 2(x \ln x - x) + c$$

$$= x(\ln x)^2 - 2x \ln x + 2x + c$$

$$= \pi \int_1^3 (\ln x)^2 dx = \left[x(\ln x)^2 - 2x \ln x + 2x \right]_1^3$$

$$= \pi \left[\left(3(\ln 3)^2 - 6 \ln 3 + 6 \right) - \left(\cancel{1(\ln 1)^2} - \cancel{2(\ln 1)} + 2 \right) \right]$$

$$= \pi \left[3(\ln 3)^2 - 6 \ln 3 + 4 \right]$$

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5) $\quad y^2 = \frac{4x+3}{(x+2)(2x-1)}$

$\pi \int y^2 dx \qquad$ too easy!

7) $\text{Vol} = \pi \int y^2 dx = \pi \int_{-1}^{2} \frac{100}{9(5+2x)^2} dx$

Let $u = 5+2x$

$\frac{du}{dx} = 2$

$du = 2dx$

$\frac{1}{2}du = dx$

$x = -1 \quad u = 3$

$x = 2 \quad u = 9$

$= \pi \int_{3}^{9} \frac{50}{9u^2} du$

$= \frac{50\pi}{9} \left[-\frac{1}{u} \right]_{3}^{9}$

$= \frac{50\pi}{9} \left[-\frac{1}{9} - -\frac{1}{3} \right]$

$= \frac{100\pi}{81}$

7b) $\pi \int \frac{400}{9(5+x)^2} dx - \pi \int \frac{100}{9(5+2x)^2} dx$

$= \frac{400\pi}{81} - \frac{100\pi}{81}$

$= \frac{300\pi}{81} = \frac{100\pi}{27}$

Exercise 4B

1b) $x = \sqrt{y}\, e^{y^2}$ between $y=0$ and $y=1$

$$\pi \int x^2 \, dy = \pi \int_0^1 y e^{2y^2} \, dy$$

By inspection $= \pi \left[\frac{1}{4} e^{2y^2} \right]_0^1$

$$= \pi \left[\frac{1}{4} e^2 - \frac{1}{4} e^0 \right]$$

$$= \frac{\pi}{4} (e^2 - 1)$$

1d) $x = \dfrac{1}{\sqrt{y \ln y}}$ between $y = e^4$ and $y = e^9$

$$\text{Vol} = \pi \int x^2 \, dy = \pi \int_{e^4}^{e^9} \frac{1}{y \ln y} \, dy$$

$$= \pi \int_{e^4}^{e^9} \frac{\frac{1}{y}}{\ln y} \, dy$$

$$= \pi \Big[\ln(\ln y) \Big]_{e^4}^{e^9}$$

$$= \pi \left[\ln(\ln e^9) - \ln(\ln e^4) \right]$$

$$= \pi \left[\ln 9 - \ln 4 \right]$$

$$= \pi \ln\left(\frac{9}{4}\right)$$

2d) $y = \cos^{-1}\sqrt{x}$ between $y=0$ and $y=\frac{\pi}{2}$

$$\text{Vol} = \pi \int x^2 dy$$

$$\cos y = \sqrt{x}$$

$$\cos^2 y = x$$

$$\cos 2y = 2\cos^2 y - 1$$

$$\frac{1+\cos 2y}{2} = \cos^2 y$$

$$\frac{1+\cos 2y}{2} = x$$

$$\frac{1 + 2\cos 2y + \cos^2 2y}{4} = x^2$$

$$\frac{1 + 2\cos 2y + \frac{1+\cos 4y}{2}}{4} = x^2$$

$$\frac{1}{4}\left[1 + 2\cos 2y + \frac{1}{2} + \frac{1}{2}\cos 4y\right] = x^2$$

$$\frac{1}{8}\left[2 + 4\cos 2y + 1 + \cos 4y\right] = x^2$$

$$\frac{1}{8}\left[3 + 4\cos 2y + \cos 4y\right] = x^2$$

$$\text{Vol} = \frac{\pi}{8}\int_0^{\frac{\pi}{2}} (3 + 4\cos 2y + \cos 4y)\, dy$$

$$= \frac{\pi}{8}\left[3y + 2\sin 2y + \frac{1}{4}\sin 4y\right]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{8}\left[\frac{3\pi}{2} + 0 + 0 - (0 + 0 - 0)\right] = \frac{3\pi^2}{16}$$

Q5 $\quad y = 3\ln(x-1) \qquad \text{Vol} = \pi\int_0^5 x^2\,dy$

$$\frac{y}{3} = \ln(x-1)$$

$$e^{\frac{y}{3}} = x-1$$

$$e^{\frac{y}{3}}+1 = x$$

$$e^{\frac{2y}{3}}+1+2e^{y/3} = x^2$$

$$\text{Vol} = \pi\int_0^5 \left(e^{\frac{2y}{3}}+2e^{\frac{y}{3}}+1\right)dy$$

$$= \pi\left[\frac{3}{2}e^{\frac{2y}{3}}+6e^{y/3}+y\right]_0^5$$

$$= \pi\left[\frac{3}{2}e^{\frac{10}{3}}+6e^{5/3}+5-\left(\frac{3}{2}+6+0\right)\right]$$

$$= \pi\left[\frac{3}{2}e^{10/3}+6e^{5/3}-\frac{5}{2}\right]$$

$$= \frac{\pi}{2}\left[3e^{10/3}+12e^{5/3}-5\right]$$

Homework

Even numbered questions Exercises 4A and 4B