

$$a \frac{dy}{dx} + by = 0$$

$$a \frac{dy}{dx} = -by$$

$$\int \frac{1}{y} dy = - \int \frac{b}{a} dx$$

$$\ln y = -\frac{b}{a}x + c$$

$$y = e^{-\frac{b}{a}x + c}$$

$$y = e^{-\frac{b}{a}x} \cdot e^c$$

$$y = Ae^{-\frac{b}{a}x}$$


---

Separation  
of

Variables

$$a \frac{dy}{dx} + by = 0$$

$$\frac{dy}{dx} + \frac{b}{a}y = 0$$

$$e^{\frac{b}{a}x} \frac{dy}{dx} + e^{\frac{b}{a}x} \frac{b}{a}y = 0$$

$$\frac{d}{dx} \left( e^{\frac{b}{a}x} y \right) = 0$$

$$e^{\frac{b}{a}x} y = C$$

$$y = \frac{C}{e^{\frac{b}{a}x}}$$

$$y = C e^{-\frac{b}{a}x}$$


---

I. F.  $e^{\int \frac{b}{a} dx}$   
 $= e^{\frac{b}{a}x}$

$$a \frac{d^2y}{dx^2} + b \frac{dy}{dx} + cy = 0$$

Auxiliary Eqn is  $am^2 + bm + c = 0$

The nature of the roots of this equation determine the form of the general solution

$$b^2 > 4ac \quad \text{real roots } \alpha, \beta \quad y = A e^{\alpha x} + B e^{\beta x}$$

$$b^2 = 4ac \quad \alpha \text{ repeated root} \quad y = (A + Bx)e^{\alpha x}$$

$$b^2 < 4ac \quad \text{complex roots} \quad p \pm iq \quad y = e^{px} (A \cos qx + B \sin qx)$$

### Exercise 7B

1 a)  $\frac{d^2y}{dx^2} + 5 \frac{dy}{dx} + 6y = 0$        $m^2 + 5m + 6 = 0$   
 $(m+2)(m+3) = 0$   
 $m = -2, m = -3$

Solution  $y = A e^{-2x} + B e^{-3x}$

1 b)  $15 \frac{d^2y}{dx^2} - 7 \frac{dy}{dx} - 2y = 0$        $15m^2 - 7m - 2 = 0$   
 $(5m+1)(3m-2) = 0$   
 $m = -\frac{1}{5}, m = \frac{2}{3}$

$y = A e^{-\frac{1}{5}x} + B e^{\frac{2}{3}x}$

---