

$$Ex 3C Q5d \quad \frac{d}{dx} \arctan(x+1)$$

$$\text{Let } y = \cot^{-1}(x+1)$$

$$\cot y = x+1$$

$$-\csc^2 y \frac{dy}{dx} = 1$$

$$\begin{aligned}\frac{dy}{dx} &= -\frac{1}{\csc^2 y} \\ &= -\frac{1}{1 + \cot^2 y} \\ &= -\frac{1}{1 + (1+x)^2}\end{aligned}$$

Integrating with Inverse Trigonometric Functions

$$\begin{aligned}1) \quad \int \frac{1}{a^2+x^2} dx &\quad \text{Let } x = a \tan \theta \\ &\quad \frac{dx}{d\theta} = a \sec^2 \theta \\ &\quad dx = a \sec^2 \theta d\theta \\ &= \int \frac{a \sec^2 \theta d\theta}{a^2 + a^2 \tan^2 \theta} \\ &= \int \frac{a \sec^2 \theta d\theta}{a^2(1 + \tan^2 \theta)} &= \int \frac{1}{a} \cdot 1 d\theta \\ &= \frac{\theta}{a} + C\end{aligned}$$

$$\text{since } x = a \tan \theta$$

$$\frac{x}{a} = \tan \theta$$

$$\tan \frac{x}{a} = \theta$$

$$= \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + C$$

$$3) \text{a)} \int \frac{3}{\sqrt{4-x^2}} dx = 3 \sin^{-1}\left(\frac{x}{2}\right) + C$$

$$\text{b)} \int \frac{4}{5+x^2} dx = \frac{4}{\sqrt{5}} \tan^{-1}\left(\frac{x}{\sqrt{5}}\right) + C$$

$$\text{c)} \int \frac{1}{\sqrt{25-x^2}} dx = \sin^{-1}\left(\frac{x}{5}\right) + C$$

$$\text{d)} \int \frac{1}{\sqrt{x^2-2}} = \operatorname{arccosh}\left(\frac{x}{\sqrt{2}}\right) + C$$

$$\begin{aligned} 4) \int \frac{1}{4+3x^2} dx &= \int \frac{1}{3\left(\frac{4}{3}+x^2\right)} dx \\ &= \frac{1}{3} \int \frac{1}{\left(\frac{2}{\sqrt{3}}\right)^2+x^2} dx \\ &= \frac{1}{3} \cdot \frac{1}{\frac{2}{\sqrt{3}}} \tan^{-1}\left(\frac{x}{\frac{2}{\sqrt{3}}}\right) + C \\ &= \frac{1}{2\sqrt{3}} \tan^{-1}\left(\frac{\sqrt{3}x}{2}\right) + C \end{aligned}$$

$$8) f(x) = \frac{2+3x}{1+3x^2}$$

$$\int f(x) dx = \int \frac{2}{1+3x^2} dx + \int \frac{3x}{1+3x^2} dx$$

$$\begin{aligned}
 &= \int \frac{2}{3(\frac{1}{3}+x^2)} dx + \frac{1}{2} \ln(1+3x^2) + C \\
 &\quad \frac{2}{3} \int \left(\frac{1}{\left(\frac{1}{\sqrt{3}}+x^2\right)} \right) dx + \frac{1}{2} \ln(1+3x^2) + C \\
 &\quad \frac{2}{3} \sqrt{3} \tan^{-1}(x\sqrt{3}) + \frac{1}{2} \ln(1+3x^2) + C
 \end{aligned}$$

9) $f(x) = \frac{2x-1}{\sqrt{2-x^2}}$

$$\int f(x) dx = \int \frac{2x}{\sqrt{2-x^2}} dx - \int \frac{1}{\sqrt{2-x^2}} dx$$

Let $u = 2-x^2$

$$\begin{aligned}
 \frac{du}{dx} &= -2x \\
 du &= -2x dx \\
 -dx &= 2x dx
 \end{aligned}$$

$$\begin{aligned}
 &- \int \frac{du}{\sqrt{u}} \\
 &= -2\sqrt{u} \\
 &\approx -2\sqrt{2-x^2} \quad - \sin^{-1}\left(\frac{x}{\sqrt{u}}\right)
 \end{aligned}$$

14)

$$\int_0^{\frac{1}{2}} \frac{x^2}{\sqrt{1-4x^2}} dx$$

Let $x = \frac{1}{2} \sin \theta$
 $\frac{dx}{d\theta} = \frac{1}{2} \cos \theta$
 $dx = \frac{1}{2} \cos \theta d\theta$

$$\begin{aligned}
 &= \int_0^{\frac{\pi}{6}} \frac{\frac{1}{4} \sin^2 \theta \cdot \frac{1}{2} \cos \theta d\theta}{\cos \theta} \quad \begin{array}{ll} x = \frac{1}{4} & \theta = \frac{\pi}{6} \\ x = 0 & \theta = 0 \end{array} \\
 &\quad \int_0^{\frac{\pi}{6}} \frac{\frac{1}{8} \sin^2 \theta d\theta}{\cos \theta} \quad (\cos 2\theta = 1 - 2\sin^2 \theta) \\
 &\quad \frac{1}{8} \int_0^{\frac{\pi}{6}} \frac{1 - \cos 2\theta}{2} d\theta \quad 2\sin^2 \theta = 1 - \cos 2\theta \\
 &\quad \frac{1}{16} \int_0^{\frac{\pi}{6}} (1 - \cos 2\theta) d\theta \quad \sin^2 \theta = \frac{1 - \cos 2\theta}{2} \\
 &= \frac{1}{16} \left[\theta - \frac{1}{2} \sin 2\theta \right]_0^{\frac{\pi}{6}} \\
 &= \frac{1}{16} \left[\frac{\pi}{6} - \frac{1}{2} \sin \frac{\pi}{3} - (0 - 0) \right] \\
 &= \frac{1}{16} \left[\frac{\pi}{6} - \frac{\sqrt{3}}{4} \right] \\
 &= \frac{1}{16 \times 12} \left[\frac{12\pi}{2} - \frac{12\sqrt{3}}{4} \right] \\
 &= \frac{1}{192} \left[2\pi - 3\sqrt{3} \right]
 \end{aligned}$$