

Edexcel FP3

Hyperbolic Functions

Leave
blank

3. (a) Starting from the definitions of $\sinh x$ and $\cosh x$ in terms of exponentials, prove that

$$\cosh 2x = 1 + 2\sinh^2 x$$

(3)

- (b) Solve the equation

$$\cosh 2x - 3 \sinh x = 15,$$

giving your answers as exact logarithms.

(5)



5. The curve C_1 has equation $y = 3\sinh 2x$, and the curve C_2 has equation $y = 13 - 3e^{2x}$.
- (a) Sketch the graph of the curves C_1 and C_2 on one set of axes, giving the equation of any asymptote and the coordinates of points where the curves cross the axes. **(4)**
- (b) Solve the equation $3\sinh 2x = 13 - 3e^{2x}$, giving your answer in the form $\frac{1}{2} \ln k$, where k is an integer. **(5)**



114

[illegible]

(a) Show that $f(x) = \frac{1}{2}(e^x + 9e^{-x})$

(2)

(b) solve $f(x) = 5$ (4)

(c) show that $\int_{\frac{1}{2}\ln 3}^{\ln 3} \frac{1}{5\cosh x - 4\sinh x} dx = \frac{\pi}{18}$ (5)



7.

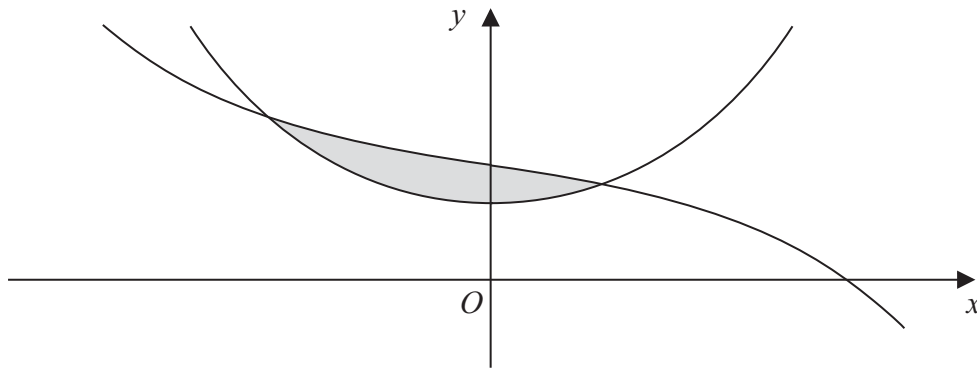


Figure 1

The curves shown in Figure 1 have equations

$$y = 6 \cosh x \quad \text{and} \quad y = 9 - 2 \sinh x$$

- (a) Using the definitions of $\sinh x$ and $\cosh x$ in terms of e^x , find exact values for the x -coordinates of the two points where the curves intersect.

(6)

The finite region between the two curves is shown shaded in Figure 1.

- (b) Using calculus, find the area of the shaded region, giving your answer in the form $a \ln b + c$, where a , b and c are integers.

(6)



Leave
blank

Question 7 continued



Further Pure Mathematics FP3

Candidates sitting FP3 may also require those formulae listed under Further Pure Mathematics FP1, and Core Mathematics C1–C4.

Vectors

The resolved part of \mathbf{a} in the direction of \mathbf{b} is $\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|}$

The point dividing AB in the ratio $\lambda : \mu$ is $\frac{\mu \mathbf{a} + \lambda \mathbf{b}}{\lambda + \mu}$

Vector product: $\mathbf{a} \times \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \sin \theta \hat{\mathbf{n}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \mathbf{b} \cdot (\mathbf{c} \times \mathbf{a}) = \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})$$

If A is the point with position vector $\mathbf{a} = a_1 \mathbf{i} + a_2 \mathbf{j} + a_3 \mathbf{k}$ and the direction vector \mathbf{b} is given by $\mathbf{b} = b_1 \mathbf{i} + b_2 \mathbf{j} + b_3 \mathbf{k}$, then the straight line through A with direction vector \mathbf{b} has cartesian equation

$$\frac{x - a_1}{b_1} = \frac{y - a_2}{b_2} = \frac{z - a_3}{b_3} (= \lambda)$$

The plane through A with normal vector $\mathbf{n} = n_1 \mathbf{i} + n_2 \mathbf{j} + n_3 \mathbf{k}$ has cartesian equation

$$n_1 x + n_2 y + n_3 z + d = 0 \text{ where } d = -\mathbf{a} \cdot \mathbf{n}$$

The plane through non-collinear points A , B and C has vector equation

$$\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a}) + \mu(\mathbf{c} - \mathbf{a}) = (1 - \lambda - \mu)\mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$$

The plane through the point with position vector \mathbf{a} and parallel to \mathbf{b} and \mathbf{c} has equation

$$\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$$

The perpendicular distance of (α, β, γ) from $n_1 x + n_2 y + n_3 z + d = 0$ is $\frac{|n_1 \alpha + n_2 \beta + n_3 \gamma + d|}{\sqrt{n_1^2 + n_2^2 + n_3^2}}$.

Hyperbolic functions

$$\cosh^2 x - \sinh^2 x = 1$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\operatorname{arcosh} x = \ln \left\{ x + \sqrt{x^2 - 1} \right\} \quad (x \geq 1)$$

$$\operatorname{arsinh} x = \ln \left\{ x + \sqrt{x^2 + 1} \right\}$$

$$\operatorname{artanh} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right) \quad (|x| < 1)$$

Conics

	Ellipse	Parabola	Hyperbola	Rectangular Hyperbola
Standard Form	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$y^2 = 4ax$	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$xy = c^2$
Parametric Form	$(a \cos \theta, b \sin \theta)$	$(at^2, 2at)$	$(a \sec \theta, b \tan \theta)$ $(\pm a \cosh \theta, b \sinh \theta)$	$\left(ct, \frac{c}{t} \right)$
Eccentricity	$e < 1$ $b^2 = a^2(1 - e^2)$	$e = 1$	$e > 1$ $b^2 = a^2(e^2 - 1)$	$e = \sqrt{2}$
Foci	$(\pm ae, 0)$	$(a, 0)$	$(\pm ae, 0)$	$(\pm\sqrt{2}c, \pm\sqrt{2}c)$
Directrices	$x = \pm \frac{a}{e}$	$x = -a$	$x = \pm \frac{a}{e}$	$x + y = \pm\sqrt{2}c$
Asymptotes	none	none	$\frac{x}{a} = \pm \frac{y}{b}$	$x = 0, y = 0$

Differentiation

$f(x)$	$f'(x)$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arccos x$	$-\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\operatorname{sech}^2 x$
$\operatorname{arsinh} x$	$\frac{1}{\sqrt{1+x^2}}$
$\operatorname{arcosh} x$	$\frac{1}{\sqrt{x^2-1}}$
$\operatorname{artanh} x$	$\frac{1}{1-x^2}$

Integration (+ constant; $a > 0$ where relevant)

$f(x)$	$\int f(x) \, dx$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\ln \cosh x$
$\frac{1}{\sqrt{a^2-x^2}}$	$\arcsin \left(\frac{x}{a} \right) \quad (x < a)$
$\frac{1}{a^2+x^2}$	$\frac{1}{a} \arctan \left(\frac{x}{a} \right)$
$\frac{1}{\sqrt{x^2-a^2}}$	$\operatorname{arcosh} \left(\frac{x}{a} \right), \quad \ln \{x + \sqrt{x^2-a^2}\} \quad (x > a)$
$\frac{1}{\sqrt{a^2+x^2}}$	$\operatorname{arsinh} \left(\frac{x}{a} \right), \quad \ln \{x + \sqrt{x^2+a^2}\}$
$\frac{1}{a^2-x^2}$	$\frac{1}{2a} \ln \left \frac{a+x}{a-x} \right = \frac{1}{a} \operatorname{artanh} \left(\frac{x}{a} \right) \quad (x < a)$
$\frac{1}{x^2-a^2}$	$\frac{1}{2a} \ln \left \frac{x-a}{x+a} \right $

Arc length

$$s = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx \quad (\text{cartesian coordinates})$$

$$s = \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt \quad (\text{parametric form})$$

Surface area of revolution

$$\begin{aligned} S_x &= 2\pi \int y \, ds = 2\pi \int y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx \\ &= 2\pi \int y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt \end{aligned}$$

Further Pure Mathematics FP1

Candidates sitting FP1 may also require those formulae listed under Core Mathematics C1 and C2.

Summations

$$\sum_{r=1}^n r^2 = \frac{1}{6} n(n+1)(2n+1)$$

$$\sum_{r=1}^n r^3 = \frac{1}{4} n^2 (n+1)^2$$

Numerical solution of equations

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Conics

	Parabola	Rectangular Hyperbola
Standard Form	$y^2 = 4ax$	$xy = c^2$
Parametric Form	$(at^2, 2at)$	$\left(ct, \frac{c}{t}\right)$
Foci	$(a, 0)$	Not required
Directrices	$x = -a$	Not required

Matrix transformations

Anticlockwise rotation through θ about O : $\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$

Reflection in the line $y = (\tan \theta)x$: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

In FP1, θ will be a multiple of 45° .

Core Mathematics C4

Candidates sitting C4 may also require those formulae listed under Core Mathematics C1, C2 and C3.

Integration (+ constant)

$$f(x) \qquad \int f(x) \, dx$$

$$\sec^2 kx \qquad \frac{1}{k} \tan kx$$

$$\tan x \qquad \ln|\sec x|$$

$$\cot x \qquad \ln|\sin x|$$

$$\operatorname{cosec} x \qquad -\ln|\operatorname{cosec} x + \cot x|, \quad \ln\left|\tan\left(\frac{1}{2}x\right)\right|$$

$$\sec x \qquad \ln|\sec x + \tan x|, \quad \ln\left|\tan\left(\frac{1}{2}x + \frac{1}{4}\pi\right)\right|$$

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Core Mathematics C3

Candidates sitting C3 may also require those formulae listed under Core Mathematics C1 and C2.

Logarithms and exponentials

$$e^{x \ln a} = a^x$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \frac{1}{2})\pi)$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\frac{f(x)}{g(x)}$	$\frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

Core Mathematics C2

Candidates sitting C2 may also require those formulae listed under Core Mathematics C1.

Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Binomial series

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \dots + \binom{n}{r} a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N})$$

$$\text{where } \binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1 \times 2} x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{1 \times 2 \times \dots \times r} x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Numerical integration

$$\text{The trapezium rule: } \int_a^b y \, dx \approx \frac{1}{2} h \{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}, \text{ where } h = \frac{b-a}{n}$$

Core Mathematics C1

Mensuration

$$\text{Surface area of sphere} = 4\pi r^2$$

$$\text{Area of curved surface of cone} = \pi r \times \text{slant height}$$

Arithmetic series

$$u_n = a + (n - 1)d$$

$$S_n = \frac{1}{2} n(a + l) = \frac{1}{2} n[2a + (n - 1)d]$$