FP2 Paper *adapted 2008

1. Solve the differential equation $\dfrac{dy}{dx} - 3y = x$

to obtain y as a function of x. (Total 5 marks)

---

3. (a) Find the general solution of the differential equation $3\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} - 2y = x^2$

(8)

(b) Find the particular solution for which, at $x = 0$, $y = 2$ and $\dfrac{dy}{dx} = 3$.(6)(Total 14 marks)

---

5. (a) Find, in terms of k, the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 3x = kt + 5,$$ where k is a constant and $t > 0$.(7)

For large values of t, this general solution may be approximated by a linear function.

(b) Given that $k = 6$, find the equation of this linear function.(2)(Total 9 marks)

---

7. (a) Show that the substitution $y = vx$ transforms the differential equation

$$\frac{dy}{dx} = \frac{x}{y} + \frac{3y}{x}, \quad x > 0, \quad y > 0 \qquad \text{(I)}$$

into the differential equation $x\dfrac{dv}{dx} = 2v + \dfrac{1}{v}$. (II) (3)

(b) By solving differential equation (II), find a general solution of differential equation (I) in the form $y = f(x)$. (7)

Given that $y = 3$ at $x = 1$, (c)find the particular solution of differential equation (I).(2)

---

9. $$(x^2+1)\frac{d^2y}{dx^2} = 2y^2 + (1-2x)\frac{dy}{dx} \qquad \text{(I)}$$

(a) By differentiating equation (I) with respect to x, show that

$$(x^2+1)\frac{d^3y}{dx^3} = (1-4x)\frac{d^2y}{dx^2} + (4y-2)\frac{dy}{dx}\,. \qquad (3)$$

Given that $y = 1$ and $\frac{dy}{dx} = 1$ at $x = 0$,

(b) find the series solution for y, in ascending powers of x, up to and including the term in x3.(4)

(c) Use your series to estimate the value of y at $x = -0.5$, giving your answer to two decimal places.(1)