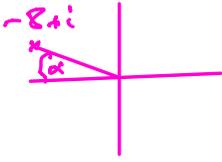


Complex Recap
Exercise 1 A

$r e^{i\alpha}$
 1) a) $-3 = 3e^{i\pi}$
 d) $-8+i$
 $\sqrt{65} e^{i 3.02}$



$\alpha = \tan^{-1} \frac{1}{8}$
 $\theta = \pi - \alpha = 3.02$
 $r = \sqrt{(-8)^2 + 1^2}$
 $r = \sqrt{65}$

g) $\sqrt{8} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$
 $= \sqrt{8} e^{i \frac{\pi}{4}}$

h) $8 \left(\cos \frac{\pi}{6} - i \sin \frac{\pi}{6} \right)$
 $= 8 \left(\cos \left(-\frac{\pi}{6} \right) + i \sin \left(-\frac{\pi}{6} \right) \right)$
 $= 8 e^{-i \frac{\pi}{6}}$

2 a) $e^{\frac{\pi i}{3}} = \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} = \frac{1}{2} + \frac{\sqrt{3}}{2} i$

d) $8 e^{\frac{\pi i}{6}} = 8 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)$
 $= 8 \left(\frac{\sqrt{3}}{2} + \frac{1}{2} i \right)$
 $= 4\sqrt{3} + 4i$

s) $e^{-\pi i} = \cos(-\pi) + i \sin(-\pi)$
 $= -1$

h) $3\sqrt{2} e^{-\frac{3\pi i}{4}} = 3\sqrt{2} \left(\cos \left(-\frac{3\pi}{4} \right) + i \sin \left(-\frac{3\pi}{4} \right) \right)$
 $3\sqrt{2} \left(-\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}} \right) = -3 - 3i$

$$3) a) e^{\frac{16\pi i}{13}} = \cos\left(-\frac{10}{13}\pi\right) + i \sin\left(-\frac{10}{13}\pi\right)$$

$$b) 4e^{\frac{17\pi i}{5}} = 4\left(\cos\left(-\frac{3\pi}{5}\right) + i \sin\left(-\frac{3\pi}{5}\right)\right)$$

$$c) 5e^{-\frac{9\pi i}{8}} = 5\left(\cos\left(\frac{7\pi}{8}\right) + i \sin\left(\frac{7\pi}{8}\right)\right)$$

$$4) e^{i\theta} = \cos\theta + i \sin\theta \quad (1)$$

$$e^{-i\theta} = \cos(-\theta) + i \sin(-\theta)$$

$$e^{-i\theta} = \cos\theta - i \sin\theta \quad (2)$$

$$(1) - (2) \quad e^{i\theta} - e^{-i\theta} = 2i \sin\theta$$

$$\frac{1}{2i}(e^{i\theta} - e^{-i\theta}) = \sin\theta$$

Exercise 1B

$$1a) e^{\frac{\pi i}{3}} \times e^{\frac{\pi i}{4}} = e^{\frac{7\pi i}{12}}$$

$$1b) \sqrt{5}e^{i\alpha} \times 3e^{3i\alpha} = 3\sqrt{5}e^{4i\alpha}$$

$$2a) \frac{2e^{\frac{7\pi i}{2}}}{8e^{\frac{9\pi i}{2}}} = \frac{1}{4}e^{-\pi i}$$

$$2b) \frac{\sqrt{3}e^{\frac{3\pi i}{7}}}{4e^{-\frac{2\pi i}{7}}} = \frac{\sqrt{3}}{4}e^{\frac{5\pi i}{7}}$$

$$3a) (\cos 2\theta + i \sin 2\theta)(\cos 3\theta + i \sin 3\theta)$$

$$= e^{i2\theta} \times e^{i3\theta}$$

$$= e^{i5\theta}$$

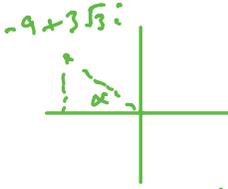
$$3c) 3 \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right) \times 2 \left(\cos \frac{\pi}{12} + i \sin \frac{\pi}{12} \right) \\ = 6 e^{\frac{i\pi}{3}}$$

$$4a) \frac{\cos 5\theta + i \sin 5\theta}{\cos 2\theta + i \sin 2\theta} = \frac{e^{i5\theta}}{e^{i2\theta}} = e^{i3\theta}$$

$$4b) \frac{\sqrt{2} \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right)}{\frac{1}{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)} = 2\sqrt{2} e^{i\frac{\pi}{4}}$$

$$4c) \frac{3 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)}{4 \left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right)} = \frac{3}{4} e^{-i\frac{\pi}{2}}$$

$$5) z = -9 + 3i\sqrt{3} \quad |w| = \sqrt{3} \\ \arg w = \frac{7\pi}{12}$$

a)  $|z| = \sqrt{(-9)^2 + (3\sqrt{3})^2} = \sqrt{81 + 27} = \sqrt{108} = 6\sqrt{3}$

$$\alpha = \tan^{-1} \left(\frac{3\sqrt{3}}{-9} \right) \\ = \tan^{-1} \frac{1}{\sqrt{3}} \\ = \frac{\pi}{6} \Rightarrow \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

$$z = 6\sqrt{3} e^{i\frac{5\pi}{6}}$$

$$b) w = \sqrt{3} e^{i\frac{7\pi}{12}}$$

$$c) zw = 18 e^{i\frac{17\pi}{12}} = 18 e^{-i\frac{7\pi}{12}}$$

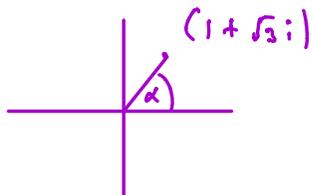
$$d) \frac{z}{w} = 6 e^{i(\frac{5\pi}{6} - \frac{7\pi}{12})} = 6 e^{i\frac{\pi}{4}}$$

$$6) \frac{(\cos 9\alpha + i \sin 9\alpha)(\cos 4\alpha + i \sin 4\alpha)}{(\cos 7\alpha + i \sin 7\alpha)}$$

$$= \frac{e^{i9\alpha} \times e^{i4\alpha}}{e^{i7\alpha}} = \frac{e^{i13\alpha}}{e^{i7\alpha}} = e^{i6\alpha}$$

$$= \cos 6\alpha + i \sin 6\alpha$$

$$7) z = 1 + i\sqrt{3} \quad \operatorname{Re}\left(\frac{z^2}{w}\right) = 0 \quad \left|\frac{z^2}{w}\right| = |z|$$



$$|z| = \sqrt{1^2 + \sqrt{3}^2} = 2$$

$$\Rightarrow |z^2| = 4$$

$$\Rightarrow |w| = 2$$

$$\alpha = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$$

$$\alpha = \frac{\pi}{3}$$

$$z = 2 e^{i\frac{\pi}{3}}$$

$$z^2 = 4 e^{i\frac{2\pi}{3}}$$

$$\text{if } \operatorname{Re}\left(\frac{z^2}{w}\right) = 0$$

$$\arg\left(\frac{z^2}{w}\right) = \pm \frac{\pi}{2}$$

$$\text{if } \arg w = \alpha$$

$$\frac{2\pi}{3} - \alpha = \pm \frac{\pi}{2}$$

$$\frac{2\pi}{3} + \frac{\pi}{2} = \alpha$$

$$\theta = \frac{7\pi}{6} \text{ or } \frac{\pi}{6}$$
$$= -\frac{5\pi}{6} \text{ or } \frac{\pi}{6}$$

$$\therefore w = 2e^{-i5\pi/6} \text{ or } w = 2e^{i\pi/6}$$

Hwks 8, 9 Start 1 c