

Inverse Functions

$$y = 2x + 3$$



$$f(x) = 2x + 3$$

$$f^{-1}(x) = \frac{x - 3}{2}$$

Algebraic Method

$$f(x) = 2x + 3$$

$$y = 2x + 3$$

Swap variables and make y the subject

$$x = 2y + 3$$

$$x - 3 = 2y$$

$$\frac{x - 3}{2} = y$$

$$f^{-1}(x) = \frac{x - 3}{2}$$

$$\text{Ex2 } f(x) = \frac{x+1}{x-2}$$

$$\text{Let } y = \frac{x+1}{x-2}$$

$$\text{Swap variables } x = \frac{y+1}{y-2}$$

$$(y-2)x = y+1$$

$$yx - 2x = y+1$$

$$y - x - y = 2x + 1$$

$$y(x-1) = 2x+1$$

$$y = \frac{2x+1}{x-1}$$

$$f^{-1}(x) = \frac{2x+1}{x-1}$$

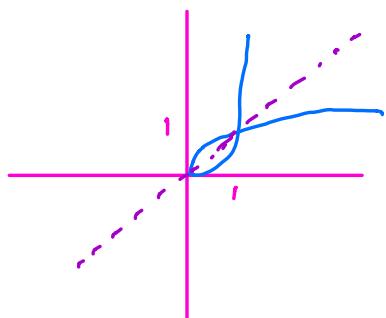
$$f(x) = x^2$$

Domain = \mathbb{R}

Range = \mathbb{R}^+

$$f^{-1}(x) = \sqrt{x}$$

(Cannot be evaluated unless domain of $f(x)$ is restricted to \mathbb{R}^+)



$$f(x) \text{ and } f^{-1}(x)$$

$f^{-1}(x)$ is a reflection of $f(x)$ in the line $y=x$

Corresponding points on $f(x)$ and $f^{-1}(x)$ have gradients that are the positive reciprocals of each other

$$\text{eg } 2 \text{ and } \frac{1}{2}$$

$$-\frac{1}{4} \text{ and } -4$$

$$\frac{2}{3} \text{ and } \frac{3}{2}$$

$$-\frac{5}{2} \text{ and } -\frac{2}{5}$$

$$\text{Eg} \quad f(x) = x^2$$

(3, 9) is on curve

Find gradient at (3, 9)

$$f'(x) = 2x$$

$$\begin{aligned}f'(3) &= 2(3) \\&= 6\end{aligned}$$

$$f^{-1}(x) = \sqrt{x}$$

(9, 3) is on curve

$$\begin{aligned}\frac{d}{dx} f^{-1}(x) &= \frac{1}{2} x^{-\frac{1}{2}} \\&= \frac{1}{2\sqrt{x}}\end{aligned}$$

$$\begin{aligned}\frac{d}{dx} f^{-1}(9) &= \frac{1}{2\sqrt{9}} \\&= \frac{1}{2(3)} \\&= \frac{1}{6}\end{aligned}$$

When asked for the gradient of a complicated inverse function, it is sometimes easier to find the gradient of the original function at the corresponding point $(x, y) \rightarrow (y, x)$ and take its reciprocal. (Positive reciprocal, not negative reciprocal as used in perpendicular gradients.)

Important Facts

1. To have an inverse a function must be 1 to 1
 2. If $f(x)$ and $f^{-1}(x)$ are inverse functions then the DOMAIN of $f(x)$ is the RANGE of $f^{-1}(x)$
The RANGE of $f(x)$ is the DOMAIN of $f^{-1}(x)$
-

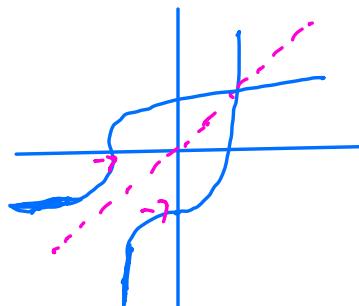
Exercise 2D

1d) $f: x \mapsto x^3 - 7$ Range = \mathbb{R}

Swap $y = x^3 - 7$
 $x = y^3 - 7$
 $x + 7 = y^3$
 $\sqrt[3]{x+7} = y$

$f^{-1}(x) = \sqrt[3]{x+7}$

Domain \mathbb{R}
Range \mathbb{R}



3) $g: x \mapsto 4 - x$ $\{x \in \mathbb{R}, x \geq 0\}$ domain
 $\{x \in \mathbb{R}, x \leq 4\}$ range

domain and range not identical
so g and g^{-1} cannot be identical!

5) $E(x) = x^2 - 6x + 5$ $\{x \in \mathbb{R}, x \geq 5\}$

Let $y = x^2 - 6x + 5$

Swap $x = y^2 - 6y + 5$
 $x = (y-3)^2 + 5 - 9$
 $x = (y-3)^2 - 4$

*

$$\begin{aligned}
 x+4 &= (y-3)^2 \\
 \sqrt{x+4} &= |y-3| \\
 3 + \sqrt{x+4} &= |y|
 \end{aligned}$$

$$7) \quad h(x) = \frac{2x+1}{x-2} \quad \left\{ x \in \mathbb{R}, x \neq 2 \right\}$$

a) $\lim_{x \rightarrow 2} h(x) \rightarrow \infty$

$$\lim_{x \rightarrow 2^+} h(x) \rightarrow \infty$$

$$\lim_{x \rightarrow 2^-} h(x) \rightarrow -\infty$$

$$h(7) = \frac{2(7)+1}{7-2} = \frac{15}{5} = 3$$

$$\therefore h'(3) = 7$$

Homework Exercise 2)

2, 4, 6, 8, 10