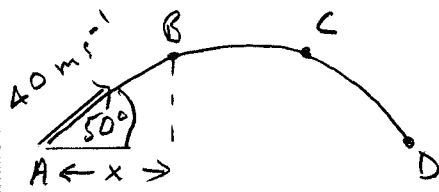


①

MEI MI JAN 2005 Q7

PROJECTILES

7)



i) $x = (40 \cos 50^\circ)t$

$$y = (40 \sin 50^\circ)t - 4.9t^2$$

ii) At D, $y=0 \Rightarrow 0 = (40 \sin 50^\circ)t - 4.9t^2$

$$0 = t(40 \sin 50^\circ - 4.9t)$$

At D $4.9t = 40 \sin 50^\circ \Rightarrow t = \frac{40 \sin 50^\circ}{4.9} = 6.253 \text{ s to 3dp}$

$$\text{Range} = AD = (40 \cos 50^\circ) \times 6.253 = 160.77 \text{ m} = 161 \text{ m to 3 s.f.}$$

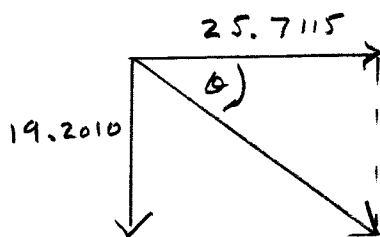
iii) At B, $x=30 \Rightarrow 30 = (40 \cos 50^\circ)t \Rightarrow t = \frac{30}{40 \cos 50^\circ} = 1.167 \text{ s}$

Time to reach B = 1.167 s, By symmetry time C to D also 1.167 s

$$\therefore \text{time A to C} = \text{time A to D} - \text{time C to D}$$

$$= 6.253 - 1.167 = 5.086 \text{ s}$$

iv)



$$v_{2c} = 40 \cos 50^\circ = 25.7115 \text{ m s}^{-1}$$

$$v_y = 40 \sin 50^\circ - gt$$

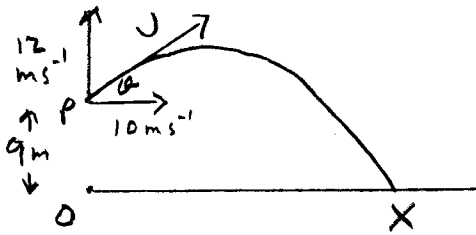
$$v_y = 40 \sin 50^\circ - 9.8 \times 5.086$$

$$= -19.2010 \text{ m s}^{-1}$$

$$\tan \alpha = \frac{19.2010}{25.7115}$$

$$\alpha = 36.8^\circ$$

Direction of motion 36.8° below horizontal



Take $g = 10 \text{ ms}^{-2}$

i) $U = \sqrt{10^2 + 12^2} = 15.6 \text{ ms}^{-1}$
to 3 s.f.

$\theta = \tan^{-1}\left(\frac{12}{10}\right) = 50.2^\circ$

Vertical

ii) $y = u_y t - \frac{1}{2} g t^2 + 9$

$y = 12t - 5t^2 + 9$

$y = 9 + 12t - 5t^2$

Horizontal

$x = 10t$

iii)

$v_y^2 = u_y^2 - 2g(y-9)$

At max height $v_y = 0$

$0 = 12^2 - 20(y-9)$

$20(y-9) = 144$

$y-9 = \frac{144}{20} = 7.2 \text{ m}$

$y-9$ is height above point of projection = 7.2 m

iv)

At X, $y = 0 \Rightarrow 0 = 9 + 12t - 5t^2$ By calc $t = 3 \text{ s}$

$OX = 10t = 10 \times 3 = 30 \text{ m}$

v)

For B, $x = (20 \cos 60)t = 10t$ same as for A

vi)

For B $y = (20 \sin 60)t - \frac{1}{2} g t^2 = 10\sqrt{3}t - 5t^2$

vii)

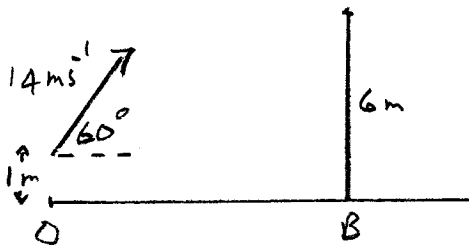
Horizontal displacements always equal so collide when vertical displacements are also equal

$\Rightarrow 9 + 12t - 5t^2 = 10\sqrt{3}t - 5t^2$

$\Rightarrow 9 = (10\sqrt{3} - 12)t$

$\Rightarrow t = \frac{9}{10\sqrt{3} - 12} = 1.69 \text{ s} = 1.7 \text{ s to 2 s.f.}$

8)



$$i) \quad x = (14 \cos 60^\circ)t = 7t$$

$$y = (14 \sin 60^\circ)t - \frac{1}{2}gt^2 + 1$$

$$y = 7\sqrt{3}t - 4.9t^2 + 1$$

ii)

A) At highest point $v_y = 0$ $v_y = 7\sqrt{3} - 9.8t \Rightarrow 0 = 7\sqrt{3} - 9.8t$

$$t = \frac{7\sqrt{3}}{9.8} = 1.24 \text{ s to 3 s.f.}$$

B) $OB = 7 \times 1.24 = 8.68 \text{ m}$

C) When $t = 1.24 \text{ s}$ $y = 7\sqrt{3} \times 1.24 - 4.9 \times 1.24^2 + 1 = 8.50 \text{ m to 3 s.f.}$

Wall is 6m high so clearance = $8.5 - 6 = 2.5 \text{ m}$

iii) $x = 7t \Rightarrow t = \frac{x}{7}$

$$y = 7\sqrt{3}t - 4.9t^2 + 1$$

so $y = 7\sqrt{3}\left(\frac{x}{7}\right) - \frac{4.9x^2}{49} + 1$

$$y = \sqrt{3}x - 0.1x^2 + 1$$

iv) Find when height = 6m $6 = 7\sqrt{3}t - 4.9t^2 + 1$

$$4.9t^2 - 7\sqrt{3}t + 5 = 0$$

$$t = 0.523 \text{ or } t = 1.951$$

When $t = 1.951$, $x = 7 \times 1.951 = 13.66 \text{ m}$

Girl needs to move away $13.66 - 8.68 = 4.98 \text{ m}$

then stone will be 6m high when it reaches wall

so $d = 4.98 \text{ m}$

5)



$$v^2 = u^2 + 2as \quad v_y^2 = u_y^2 - 19.6y$$

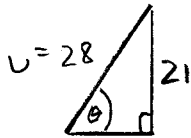
$$\text{At max height } v_y = 0 \quad 0 = u_y^2 - 19.6 \times 22.5$$

$$\sqrt{19.6 \times 22.5} = u_y$$

$$21 \text{ ms}^{-1} = u_y$$

\therefore vertical component of initial velocity = 21 ms^{-1}

ii)



$$\theta = \sin^{-1}\left(\frac{21}{28}\right) = 48.6^\circ$$

Projected 48.6° above horizontal

iii)

$$y = 21t - 4.9t^2$$

Lands when $y = 0$

$$0 = 21t - 4.9t^2$$

$$0 = t(21 - 4.9t)$$

$$t = \frac{21}{4.9} = \frac{30}{7} \text{ s}$$

$$x = (28 \cos \theta) t$$

$$x = 28 \cos 48.6 \times \frac{30}{7} = 79.4 \text{ m}$$

Horizontal range = 79.4 m