Paper 4761	Name Mechanics 1	Session Jan	Year 2005	

Solutions and mark scheme

Q 7		mark		
(i)	Horiz $(40\cos 50)t$	B1		
	Vert $(40\sin 50)t - 4.9t^2$	M1	Use of $s = ut + 0.5at^2$ with $a = \pm 9.8$ or ± 10. Allow u = 40. Condone s $\leftrightarrow$ c.	
		A1	Any form	3
(ii)	Need $(40\sin 50)t - 4.9t^2 = 0$	M1	Equating **their** y to zero. Allow quadratic y only	
	so $t = \dfrac{40\sin 50}{4.9}$	M1	Dep on 1st M1. Attempt to solve.	
	= 6.2534… so 6.253 s (3 d. p.)	E1	Clearly shown [**or** M1 (allow u = 40 and s $\leftrightarrow$ c) A1 time to greatest height; E1]	
	Range is $(40\cos 50)\times 6.2534...$	M1	Use of **their** horiz expression	
	= 160.78… so 161 m (3 s. f.)	A1	Any reasonable accuracy	5
(iii)	Time AB is given by $(40\cos 50)T = 30$ so T = 1.16679… so 1.17 s	M1 A1	Equating **their** linear x to 30.	
	then **either** By symmetry, time AC is time AD – time AB	M1	Symmetry need not be explicit. Method may be implied. Any valid method using symmetry.	
	so time AC is 6.2534… $-\dfrac{30}{40\cos 50}$ = 5.086…. so 5.09 s (3 s. f.)	A1	cao	
	or height is $(40\sin 50)T - 4.9T^2$ and we need $(40\sin 50)t - 4.9t^2 = (40\sin 50)T - 4.9T^2$ solved for larger root	M1	Complete method to find time to second occasion at that height	
	i.e. solve $4.9t^2 - (40\sin 50)t + 29.08712... = 0$ for larger root giving 5.086…	A1	cao	4
(iv)	$\dot{x} = 40\cos 50$	B1	Must be part of a method using velocities.	
	$\dot{y} = 40\sin 50 - 9.8\times 5.086...$	M1 A1	Use of vert cpt of vel Allow only sign error. FT use of **their** 5.086..	
	Need $\arctan\dfrac{\dot{y}}{\dot{x}}$	M1	May be implied. Accept $\arctan\dfrac{\dot{x}}{\dot{y}}$ but not use of $\dot{y}_0$.	
	So –36.761…° so 36.8° below horizontal (3 s.f.)	A1	Accept ±36.8 or equivalent. Condone direction not clear.	5
	total	17		

Q 7		mark		Sub
(i)	$u = \sqrt{10^2 + 12^2} = 15.62..$ $\theta = \arctan\left(\frac{12}{10}\right) = 50.1944...$ so 50.2 (3s.f.)	B1 M1 A1	Accept any accuracy 2 s. f. or better Accept $\arctan\left(\frac{10}{12}\right)$ (Or **their** $15.62\cos\theta = 10$ or **their** $15.62\sin\theta = 12$) [FT **their** 15.62 if used] [If θ found first M1 A1 for θ F1 for u] [If B0 M0 SC1 for both $u\cos\theta = 10$ and $u\sin\theta = 12$ seen]	3
(ii)	vert $12t - 0.5\times10t^2 + 9$ $= 12t - 5t^2 + 9$ (AG) horiz $10t$	M1 A1 E1 B1	Use of $s = ut + 0.5at^2$, $a = \pm9.8$ or ±10 and $u = 12$ or 15.62.. Condone $-9 = 12t - 0.5\times10t^2$, condone $y = 9 + 12t - 0.5\times10t^2$. Condone g. All correct with origin of $u = 12$ clear; accept 9 omitted Reason for 9 given. Must be clear unless $y = s_0 + ...$ used.	4
(iii)	$0 = 12^2 - 20s$ $s = 7.2$ so 7.2 m	M1 A1	Use of $v^2 = u^2 + 2as$ or equiv with $u = 12$, $v = 0$. Condone $u \leftrightarrow v$ From CWO. Accept 16.2.	2
(iv)	We require $0 = 12t - 5t^2 + 9$ Solve for t the + ve root is 3 range is 30 m	M1 M1 A1 F1	Use of y equated to 0 Attempt to solve a 3 term quadratic Accept no reference to other root. cao. FT root and **their** x. [If range split up M1 all parts considered; M1 valid method for each part; A1 final phase correct; A1]	4
(v)	Horiz displacement of B: $20\cos 60t = 10t$ Comparison with Horiz displacement of A	B1 E1	Condone unsimplified expression. Award for $20\cos60 = 10$ Comparison clear, must show $10t$ for each or explain.	2
(vi)	vertical height is $20\sin 60t - 0.5\times10t^2 = 10\sqrt{3}t - 5t^2$ (AG)	A1	Clearly shown. Accept decimal equivalence for $10\sqrt{3}$ (at least 3 s. f.). Accept $-5t^2$ and $20\sin60 = 10\sqrt{3}$ not explained.	1
(vii)	Need $10\sqrt{3}t - 5t^2 = 12t - 5t^2 + 9$ $\Rightarrow t = \frac{9}{10\sqrt{3} - 12}$ $t = 1.6915\ldots$ so 1.7 s (2 s. f.) (AG)	M1 A1 E1	Equating the **given** expressions Expression for t obtained in any form Clearly shown. Accept 3 s. f. or better as evidence. Award M1 A1 E0 for 1.7 sub in each ht	3
	total	19		

Q 8		mark		Sub
(i)	$x = 14\cos 60t$	M1	Consider motion in x direction. Need not resolve. Allow $\sin \leftrightarrow \cos$. Condone +1 seen.	
	so $x = 7t$	A1	Need not be simplified.	
	$y = 14\sin 60t - 4.9t^2 + 1$	M1	Suitable *uvast* used for y with $g = \pm 9.8, \pm 10, \pm 9.81$ soi Need not resolve. Allow $\sin \leftrightarrow \cos$.	
		A1	Allow + 1omitted. Any form and 2 s. f. Need not be simplified	
	$y = 7\sqrt{3}t - 4.9t^2 + 1$ ($y = 12.124...t - 4.9t^2 + 1$)	A1	All correct. +1 need not be justified. Accept any form and 2 s. f. Need not be simplified.	5
(ii) (A)	time taken to reach highest point $0 = 7\sqrt{3} - 9.8T$	M1	Appropriate *uvast* . Accept $u = 14$ and $\sin \leftrightarrow \cos$ and $u \leftrightarrow v$. Require $v = 0$ or equivalent. $g = \pm 9.8, \pm 10, \pm 9.81$ soi.	
	so $\frac{5\sqrt{3}}{7}$ s (1.23717…. = 1.24 s (3 s. f.))	A1	cao [If time of flight attempted, do not award M1 if twice interval obtained]	2
(B)	distance from base is $7 \times \frac{5\sqrt{3}}{7} = 5\sqrt{3}$ m (= 8.66025… so 8.66 m (3 s. f.))	M1	Use of **their** $x = 7t$ with **their** T	
		B1	FT **their** T only in $x = 7t$. Accept values rounding to 8.6 and 8.7.	2
(C)	**either** Height at this time is $H = 7\sqrt{3} \times \frac{5\sqrt{3}}{7} - 4.9 \times \left(\frac{5\sqrt{3}}{7}\right)^2 + 1$	M1	Subst in **their quadratic** y with **their** T.	
		A1	Correct subst of **their** T in **their** y which has attempts at all 3 terms. Do not accept $u = 14$.	
	= 8.5	A1		

	clearance is $8.5-6=2.5$ m **or** for height above pt of projection $0=\left(7\sqrt{3}\right)^2+2\times-9.8\times s$ s = 7.5 so clearance is $7.5-5=2.5$ m	E1 M1 A1 A1 E1	Clearly shown. Appropriate *uvast* . Accept u = 14. $g = \pm 9.8, \pm 10, \pm 9.81$ soi Attempt at vert cpt accept $\sin \leftrightarrow \cos$ .Accept sign errors but not u = 14. Clearly shown.	4
(iii)	See over			

Q 8	**continued**	**mark**		**sub**
(iii)	Elim t between $y=7\sqrt{3}t-4.9t^2+1$ and $x = 7t$ so $y=7\sqrt{3}\frac{x}{7}-4.9\left(\frac{x}{7}\right)^2+1$ so $y=\sqrt{3}x-0.1x^2+1$	M1 F1	Must see **their** $t = x/7$ fully substituted in **their** quadratic y (accept bracket errors) Accept any form correctly written. FT **their** x and 3 term quadratic y (neither using u = 14)	2
(iv)	**either** need $6=7\sqrt{3}t-4.9t^2+1$ so $4.9t^2-7\sqrt{3}t+5=0$ $t=\frac{5\left(\sqrt{3}\pm1\right)}{7}$ (0.52289…. or 1.95146…) moves by $\left(\frac{5(\sqrt{3}+1)}{7}-\frac{5\sqrt{3}}{7}\right)\times7$ $[(1.95146..-1.23717\ldots)\times7]$ = 5 m **or** using equation of trajectory with y = 6	M1 M1 A1 M1 A1	**their** quadratic y from (i) = 6, or equivalent. Dep. Attempt to solve this 3 term quadratic. (Allow u = 14). for either root Moves by $\lvert$**their** root - **their** (ii)(A)$\rvert\times7$ or equivalent. Award this for recognition of correct dist (no calc) cao [If new distance to wall found must have larger of 2 +ve roots for 3rd M and award max 4/5 for 13.66]	

	$6 = \sqrt{3}x - 0.1x^2 + 1$	M1	Equating **their** quadratic trajectory equn to 6	
	Solving $x^2 - 10\sqrt{3}x + 50 = 0$	M1	Dep. Attempt to solve this 3 term quadratic. (Allow u = 14).	
	$x = 5\left(\sqrt{3} \pm 1\right)$ (13.660… or 3.6602….)	A1	for either root	
	distance is $5\left(\sqrt{3}+1\right) - 5\sqrt{3}$	M1	distance is $\lvert$**their** root – **their**(ii)(B)$\rvert$ Award this for recognition of correct dist (no calc)	
	= 5 m	A1	Cao [If new distance to wall found must have larger of 2 + ve roots for 3rd M and award max 4/5 for 13.66]	5
				20

Q 5		mark		Sub
(i)	$0^2 = V^2 - 2\times 9.8\times 22.5$ $V = 21$ so 21 m s^{-1}	M1 E1	Use of appropriate *uvast*. Give for correct expression Clearly shown. Do not allow $v^2 = 0 + 2gs$ without explanation. Accept using $V = 21$ to show $s = 22.5$.	2
(ii)	$28\sin\theta = 21$ so $\theta = 48.59037...$	M1 A1	Attempt to find angle of projection. Allow $\sin \leftrightarrow \cos$.	2
(iii)	Time to highest point is $\frac{21}{9.8} = \frac{15}{7}$	B1	Or equivalent (time of whole flight)	
	Distance is $2\times\frac{15}{7}\times 28\times\cos(\textbf{their}\,\theta)$..	M1	Valid method for horizontal distance. Accept ½ time. Do not accept 28 used for horizontal speed or vertical speed when calculating time.	
		B1	Horizontal speed correct	
	79.3725… so 79.4 m (3 s. f.)	A1	cao. Accept answers rounding to 79 or 80. [If angle with vertical found in (ii) allow up to full marks in (iii). If $\sin \leftrightarrow \cos$ allow up to B1 B1 M0 A1] [If $u^2\sin 2\theta / g$ used then M1* Correct formula used. FT their angle. M1 Dep on *. Correct subst. FT their angle. A2 cao]	4
				8