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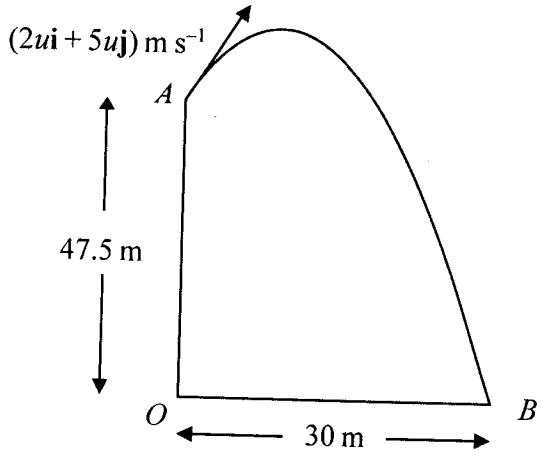

Figure 3

[*In this question, the unit vectors* **i** *and* **j** *are in a vertical plane,* **i** *being horizontal and* **j** *being vertical.*]

A particle P is projected from the point A which has position vector $47.5\mathbf{j}$ metres with respect to a fixed origin O. The velocity of projection of P is $(2u\mathbf{i} + 5u\mathbf{j})$ m s^{-1}. The particle moves freely under gravity passing through the point B with position vector $30\mathbf{i}$ metres, as shown in Figure 3.

(a) Show that the time taken for P to move from A to B is 5 s.

(6)

(b) Find the value of u.

(2)

(c) Find the speed of P at B.

(5)

a) $\underline{r} = \underline{u}t + \frac{1}{2}\underline{a}t^2 + \begin{pmatrix} 0 \\ 47.5 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2u \\ 5u \end{pmatrix} t + \begin{pmatrix} 0 \\ -4.9 \end{pmatrix} t^2 + \begin{pmatrix} 0 \\ 47.5 \end{pmatrix}$

At B, $x = 30$

$30 = 2ut + 0 + 0$

$\frac{30}{2t} = u$

$\frac{15}{t} = u$

N 2 5 2 5 2 A 0 1 6 2 4

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Question 6 continued

At B, $y = 0$ $\quad 0 = 5ut - 4.9t^2 + 47.5$

Sub for u $\quad 0 = 5 \times \frac{15}{t} \times t - 4.9t^2 + 47.5$

$0 = 75 - 4.9t^2 + 47.5$

$4.9t^2 = 122.5$

$t^2 = 25 \quad \Rightarrow t = 5\,s$

b) $u = \frac{15}{t} = \frac{15}{5} = 3 \qquad u = 3$

c) vertical component $\quad V_y = u_y + at$

At B, $\quad V_y = 5 \times 3 - 9.8 \times 5 = -34$

Horizontal component

$V_x = 2 \times 3 = 6$

Speed $= \sqrt{V_x^2 + V_y^2}$

$= \sqrt{6^2 + (-34)^2}$

$= \sqrt{1192}$

$= 34.5\,ms^{-1}$

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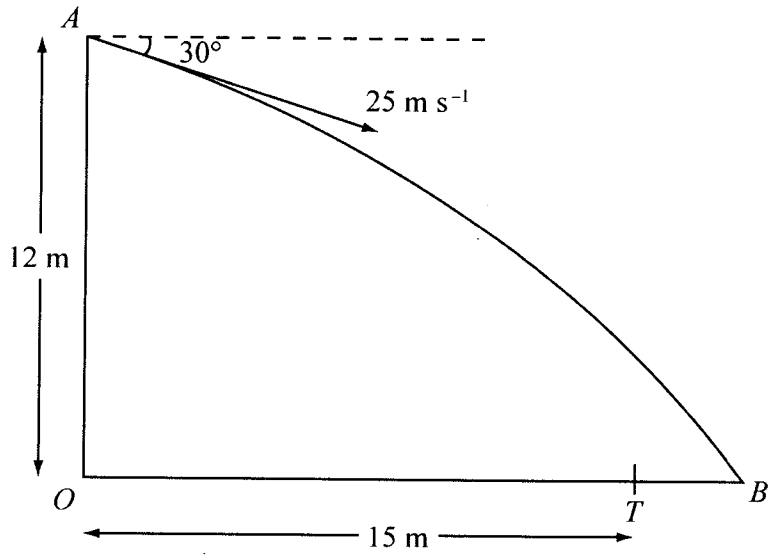

Figure 4

A ball is thrown from a point A at a target, which is on horizontal ground. The point A is 12 m above the point O on the ground. The ball is thrown from A with speed 25 m s^{-1} at an angle of 30° below the horizontal. The ball is modelled as a particle and the target as a point T. The distance OT is 15 m. The ball misses the target and hits the ground at the point B, where OTB is a straight line, as shown in Figure 4. Find

(a) the time taken by the ball to travel from A to B,

(5)

(b) the distance TB.

(4)

The point X is on the path of the ball vertically above T.

(c) Find the speed of the ball at X.

(5)

a) $\updownarrow$ $s = ut + \frac{1}{2}at^2 + s_0$ $\quad y = (-25\sin 30)t - 4.9t^2 + 12$

At B, $y = 0$ $\quad 0 = -\frac{25}{2}t - 4.9t^2 + 12$

$4.9t^2 + 12.5t - 12 = 0$

$t = 0.743\text{ s}$ $\quad \cancel{t = -3.294}$

H 2 9 4 9 8 A 0 2 0 2 4

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Question 7 continued

b) $\leftrightarrow$ $x = (25\cos 30^\circ)t$

At B, $t = 0.743\,s$ $\quad x = 25\cos 30^\circ \times 0.743$

$x = 16.1\,m$

$TB = OB - OT = 16.1 - 15 = 1.1\,m$

c) Find time when above T

$x = (25\cos 30^\circ)t$

$t = \dfrac{x}{25\cos 30^\circ} = \dfrac{15}{25\cos 30} = 0.693\,s$

$\updownarrow$ $v = u + at$

$V_y = -25\sin 30^\circ - 9.8t$

$V_y = -25\sin 30^\circ - 9.8 \times 0.693 = -19.3\,ms^{-1}$

$\leftrightarrow$ $V_x = 25\cos 30^\circ$

Speed $= \sqrt{V_x^2 + V_y^2}$

$= \sqrt{(25\cos 30^\circ)^2 + (-19.3)^2}$

$= 29.0\,ms^{-1}$

Q7

(Total 14 marks)

TOTAL FOR PAPER: 75 MARKS

END

H 2 9 4 9 8 A 0 2 3 2 4

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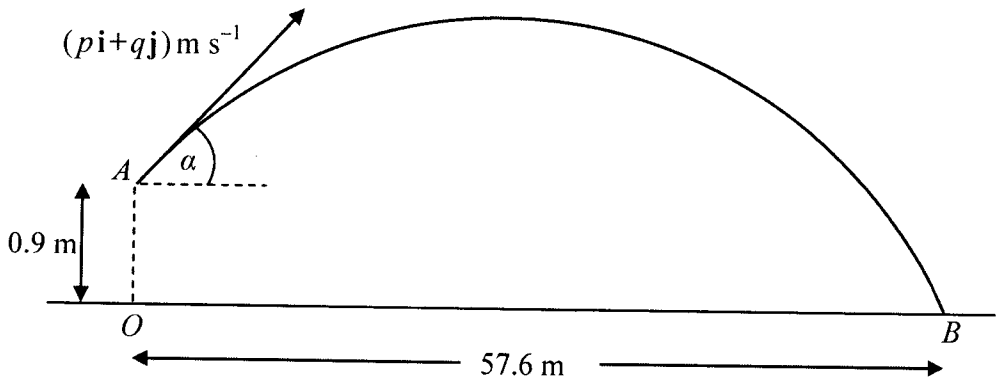

Figure 3

A cricket ball is hit from a point A with velocity of $(p\mathbf{i} + q\mathbf{j})$ m s^{-1}, at an angle α above the horizontal. The unit vectors **i** and **j** are respectively horizontal and vertically upwards. The point A is 0.9 m vertically above the point O, which is on horizontal ground.

The ball takes 3 seconds to travel from A to B, where B is on the ground and $OB = 57.6$ m, as shown in Figure 3. By modelling the motion of the cricket ball as that of a particle moving freely under gravity,

(a) find the value of p,

(2)

(b) show that $q = 14.4$,

(3)

(c) find the initial speed of the cricket ball,

(2)

(d) find the exact value of $\tan \alpha$.

(1)

(e) Find the length of time for which the cricket ball is at least 4 m above the ground.

(6)

(f) State an additional physical factor which may be taken into account in a refinement of the above model to make it more realistic.

(1)

$\begin{pmatrix} x \\ y \end{pmatrix} = \underline{u}t + \frac{1}{2}\underline{a}t^2 + \begin{pmatrix} 0 \\ 0.9 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} p \\ q \end{pmatrix} t + \begin{pmatrix} 0 \\ -4.9 \end{pmatrix} t^2 + \begin{pmatrix} 0 \\ 0.9 \end{pmatrix}$

At B, $x = 57.6$, $t = 3$

$57.6 = 3p + 0 + 0 \Rightarrow p = 19.2$

N 3 0 0 8 3 A 0 1 6 2 4

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Question 6 continued

b) At B, $y=0$, $t=3$

$0 = 3q - 4.9 \times 3^2 + 0.9$

$44.1 - 0.9 = 3q$

$\frac{43.2}{3} = q$

$q = 14.4$

c) Initial speed $= \sqrt{p^2+q^2}$

$= \sqrt{19.2^2 + 14.4^2} = 24\,ms^{-1}$

d)

$\tan\alpha = \frac{q}{p} = \frac{14.4}{19.2}$

$\tan\alpha = \frac{3}{4}$

e) Find when $y = 4m$

$4 = 14.4t - 4.9t^2 + 0.9$

$4.9t^2 - 14.4t + 3.1 = 0$

$t = 2.705$, $t = 0.234$

Above 4m between these times

$2.705 - 0.234 = 2.471$

$= 2.47s$

f)

Air resistance

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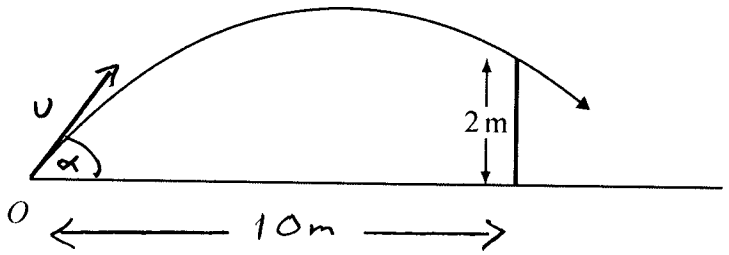

Figure 3

A child playing cricket on horizontal ground hits the ball towards a fence 10 m away. The ball moves in a vertical plane which is perpendicular to the fence. The ball just passes over the top of the fence, which is 2 m above the ground, as shown in Figure 3.

The ball is modelled as a particle projected with initial speed u m s^{-1} from point O on the ground at an angle α to the ground.

(a) By writing down expressions for the horizontal and vertical distances, from O of the ball t seconds after it was hit, show that

$$2 = 10\tan\alpha - \frac{50g}{u^2\cos^2\alpha}.$$

(6)

Given that $\alpha = 45°$,

(b) find the speed of the ball as it passes over the fence.

(6)

$\longleftrightarrow$ $x = (u\cos\alpha)t$ ①

$\updownarrow$ $y = (u\sin\alpha)t - \frac{1}{2}gt^2$ ②

from ① $t = \dfrac{x}{u\cos\alpha}$

Sub for t in ② $y = \dfrac{(u\sin\alpha)x}{u\cos\alpha} - \dfrac{gx^2}{2u^2\cos^2\alpha}$

When $x = 10$, $y = 2$

$$2 = 10\tan\alpha - \frac{50g}{u^2\cos^2\alpha}$$

N 3 4 2 7 4 A 0 1 6 2 8

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Question 6 continued

b) $\alpha = 45^\circ$

$$2 = 10\tan 45 - \frac{50g}{u^2\cos^2 45}$$

$$2 = 10 - \frac{100g}{u^2}$$

$$\frac{100g}{u^2} = 8$$

$$100g = 8u^2$$

$$\frac{100g}{8} = u^2$$

$$u = 11.068$$

$$u = 11.1\,ms^{-1}$$

$\longleftrightarrow$ $V_x = 11.1\cos 45^\circ = 7.85\,ms^{-1}$

$\updownarrow$ At fence $y = 2$

$$V_y^2 = u_y^2 - 19.6y$$

$$V_y^2 = (11.1\sin 45)^2 - 19.6 \times 2$$

$$V_y^2 = 22.405$$

Speed at fence $= \sqrt{V_x^2 + V_y^2}$

$$= \sqrt{7.85^2 + 22.405}$$

$$= 9.17\,ms^{-1}$$

Leave blank

8. [*In this question* **i** *and* **j** *are unit vectors in a horizontal and upward vertical direction respectively*]

A particle P is projected from a fixed point O on horizontal ground with velocity $u(\mathbf{i} + c\mathbf{j})\,\mathrm{m\,s^{-1}}$, where c and u are positive constants. The particle moves freely under gravity until it strikes the ground at A, where it immediately comes to rest. Relative to O, the position vector of a point on the path of P is $(x\mathbf{i} + y\mathbf{j})\,\mathrm{m}$.

(a) Show that

$$y = cx - \frac{4.9x^2}{u^2}.$$

(5)

Given that $u = 7$, $OA = R\,\mathrm{m}$ and the maximum vertical height of P above the ground is $H\,\mathrm{m}$,

(b) using the result in part (a), or otherwise, find, in terms of c,

(i) R

(ii) H.

(6)

Given also that when P is at the point Q, the velocity of P is at right angles to its initial velocity,

(c) find, in terms of c, the value of x at Q.

(6)

a) $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} u \\ cu \end{pmatrix} t + \begin{pmatrix} 0 \\ -4.9 \end{pmatrix} t^2$

$\Rightarrow \quad x = ut \quad \Rightarrow \quad t = \frac{x}{u}$

$y = cut - 4.9t^2$

sub for t $\quad y = cu\frac{x}{u} - 4.9\frac{x^2}{u^2}$

$y = cx - \frac{4.9x^2}{u^2}$

b) i) $y = 0$ when $x = R$ $\quad u = 7$

$0 = cR - \frac{4.9R^2}{7^2}$

M 3 5 1 0 2 A 0 2 4 2 8

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Question 8 continued

$0 = cR - 0.1R^2 = R(c - 0.1R)$

At A, $c - 0.1R = 0 \Rightarrow R = 10c$

ii) $y = H$ when $x = \frac{R}{2}$

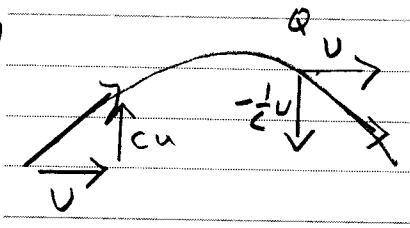
$H = \frac{cR}{2} - 4.9\frac{\left(\frac{R}{2}\right)^2}{49}$

$H = \frac{c \times 10c}{2} - 0.1 \times \frac{100c^2}{4}$

$H = 5c^2 - 2.5c^2$

$H = \frac{5c^2}{2}$

c)



Gradient at start $= c$

Gradient at Q $= -\frac{1}{c}$

So V_y at Q given by

$V_y = -\frac{1}{c}u = -\frac{7}{c}$

$\updownarrow$ $V_y = u_y - 9.8t$

$-\frac{7}{c} = 7c - 9.8t \Rightarrow 9.8t = 7c + \frac{7}{c}$

$t = \frac{\left(7c + \frac{7}{c}\right)}{9.8}$

$t = \frac{\left(c + \frac{1}{c}\right)}{1.4}$

$x = u_x t = \frac{7\left(c + \frac{1}{c}\right)}{1.4} = 5\left(c + \frac{1}{c}\right)$

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7.

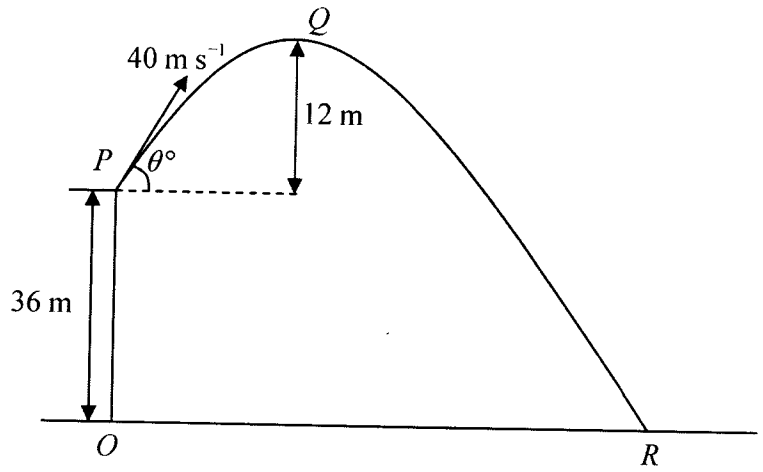

Figure 3

A ball is projected with speed 40 m s^{-1} from a point P on a cliff above horizontal ground. The point O on the ground is vertically below P and OP is 36 m. The ball is projected at an angle $\theta°$ to the horizontal. The point Q is the highest point of the path of the ball and is 12 m above the level of P. The ball moves freely under gravity and hits the ground at the point R, as shown in Figure 3. Find

(a) the value of θ,

(3)

(b) the distance OR,

(6)

(c) the speed of the ball as it hits the ground at R.

(3)

a) $v_y^2 = u_y^2 - 19.6(y-36)$

At top $v_y = 0$ $\quad 0 = (40\sin\theta)^2 - 19.6 \times 12$

$235.2 = 1600\sin^2\theta$

$\sqrt{\frac{235.2}{1600}} = \sin\theta$

$\theta = 22.5°$

b) At R, $y = 0$

$y = u_y t - 4.9t^2 + 36$

$y = (40\sin 22.5°)t - 4.9t^2 + 36$

N 3 5 3 9 1 A 0 2 4 3 2

Leave blank

Question 7 continued

At R, $y = 0$ $\quad 0 = (40\sin 22.5°)t - 4.9t^2 + 36$

$4.9t^2 - 40\sin 22.5 - 36 = 0$

By calc $\quad t = 4.69\,s \quad \cancel{t = -1.57\,s}$

$x = (40\cos 22.5°)t = (40\cos 22.5°) \times 4.69$

$x = 173\,m$ to 3 s.f.

c) At R

$V_y = u_y - 9.8t = 40\sin 22.5° - 9.8 \times 4.69$

$V_y = -30.65\,ms^{-1}$

$V_x = 40\cos 22.5°$

Speed $= \sqrt{V_x^2 + V_y^2}$

$= \sqrt{(40\cos 22.5°)^2 + (-30.65)^2}$

$= 48.0\,ms^{-1}$