

1. A particle P moves on the x -axis. The acceleration of P at time t seconds, $t \geq 0$, is $(3t + 5) \text{ m s}^{-2}$ in the positive x -direction. When $t = 0$, the velocity of P is 2 m s^{-1} in the positive x -direction. When $t = T$, the velocity of P is 6 m s^{-1} in the positive x -direction. Find the value of T .
(Total 6 marks)

2. A particle P moves along the x -axis. At time t seconds the velocity of P is $v \text{ m s}^{-1}$ in the positive x -direction, where $v = 3t^2 - 4t + 3$. When $t = 0$, P is at the origin O . Find the distance of P from O when P is moving with minimum velocity.
(Total 8 marks)

3. At time $t = 0$ a particle P leaves the origin O and moves along the x -axis. At time t seconds the velocity of P is $v \text{ m s}^{-1}$, where

$$v = 8t - t^2.$$

- (a) Find the maximum value of v .
(4)
- (b) Find the time taken for P to return to O .
(5)
(Total 9 marks)

4. A particle P moves along the x -axis in a straight line so that, at time t seconds, the velocity of P is $v \text{ m s}^{-1}$, where

$$v = \begin{cases} 10t - 2t^2, & 0 \leq t \leq 6, \\ \frac{-432}{t^2}, & t > 6. \end{cases}$$

At $t = 0$, P is at the origin O . Find the displacement of P from O when

(a) $t = 6$, (3)

(b) $t = 10$. (5)

(Total 8 marks)

5. A particle P moves on the x -axis. At time t seconds the velocity of P is v m s⁻¹ in the direction of x increasing, where v is given by

$$v = \begin{cases} 8t - \frac{3}{2}t^2, & 0 \leq t \leq 4, \\ 16 - 2t, & t > 4. \end{cases}$$

When $t = 0$, P is at the origin O .

Find

(a) the greatest speed of P in the interval $0 \leq t \leq 4$, (4)

(b) the distance of P from O when $t = 4$, (3)

(c) the time at which P is instantaneously at rest for $t > 4$, (1)

(d) the total distance travelled by P in the first 10 s of its motion. (8)

(Total 16 marks)

6. A particle P moves on the x -axis. At time t seconds, its acceleration is $(5 - 2t)\text{m s}^{-2}$, measured in the direction of x increasing. When $t = 0$, its velocity is 6 m s^{-1} measured in the direction of x increasing. Find the time when P is instantaneously at rest in the subsequent motion. (Total 6 marks)

7. A cricket ball of mass 0.5 kg is struck by a bat. Immediately before being struck, the velocity of the ball is $(-30\mathbf{i})\text{ m s}^{-1}$. Immediately after being struck, the velocity of the ball is $(16\mathbf{i} + 20\mathbf{j})\text{ m s}^{-1}$.
- (a) Find the magnitude of the impulse exerted on the ball by the bat. (4)

In the subsequent motion, the position vector of the ball is \mathbf{r} metres at time t seconds. In a model of the situation, it is assumed that $\mathbf{r} = [16t\mathbf{i} + (20t - 5t^2)\mathbf{j}]$. Using this model,

- (b) find the speed of the ball when $t = 3$. (4)
- (Total 8 marks)

8. A particle P of mass 0.4 kg is moving so that its position vector \mathbf{r} metres at time t seconds is given by

$$\mathbf{r} = (t^2 + 4t)\mathbf{i} + (3t - t^3)\mathbf{j}.$$

- (a) Calculate the speed of P when $t = 3$. (5)

When $t = 3$, the particle P is given an impulse $(8\mathbf{i} - 12\mathbf{j})\text{ N s}$.

- (b) Find the velocity of P immediately after the impulse. (3)
- (Total 8 marks)

9. A particle P moves in a horizontal plane. At time t seconds, the position vector of P is \mathbf{r} metres relative to a fixed origin O , and \mathbf{r} is given by

$$\mathbf{r} = (18t - 4t^3)\mathbf{i} + ct^2\mathbf{j},$$

where c is a positive constant. When $t = 1.5$, the speed of P is 15 m s^{-1} . Find

- (a) the value of c ,

(6)

- (b) the acceleration of P when $t = 1.5$.

(3)

(Total 9 marks)

10. At time t seconds, the velocity of a particle P is $[(4t - 7)\mathbf{i} - 5\mathbf{j}] \text{ m s}^{-1}$. When $t = 0$, P is at the point with position vector $(3\mathbf{i} + 5\mathbf{j}) \text{ m}$ relative to a fixed origin O .

- (a) Find an expression for the position vector of P after t seconds, giving your answer in the form $(a\mathbf{i} + b\mathbf{j}) \text{ m}$.

(4)

A second particle Q moves with constant velocity $(2\mathbf{i} - 3\mathbf{j}) \text{ m s}^{-1}$. When $t = 0$, the position vector of Q is $(-7\mathbf{i}) \text{ m}$.

- (b) Prove that P and Q collide.

(6)

(Total 10 marks)

11. A particle P moves on the x -axis. At time t seconds the velocity of P is $v \text{ m s}^{-1}$ in the direction of x increasing, where $v = 6t - 2t^2$. When $t = 0$, P is at the origin O . Find the distance of P from O when P comes to instantaneous rest after leaving O .

(Total 5 marks)

12. A particle P moves on the x -axis. The acceleration of P at time t seconds is $(4t - 8) \text{ m s}^{-2}$, measured in the direction of x increasing. The velocity of P at time t seconds is $v \text{ m s}^{-1}$. Given that $v = 6$ when $t = 0$, find

(a) v in terms of t ,

(4)

(b) the distance between the two points where P is instantaneously at rest.

(7)

(Total 11 marks)