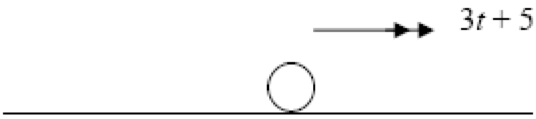
1.



$\frac{dv}{dt} = 3t + 5$

$v = \int (3t+5)\,dt$	M1 *
$v = \frac{3}{2}t^2 + 5t \quad (+c)$	A1
$t = 0 \quad v = 2 \quad \Rightarrow \quad c = 2$	B1
$v = \frac{3}{2}t^2 + 5t + 2$	
$t = T \qquad 6 = \frac{3}{2}T^2 + 5T + 2$	DM1 *
$12 = 3T^2 + 10T + 4$	
$3T^2 + 10T - 8 = 0$	
$(3T - 2)(T + 4) = 0$	M1
$T = \frac{2}{3} \quad (T = -4)$	
$\therefore T = \frac{2}{3}$ (or 0.67)	A1

[6]

2. $\frac{dv}{dt} = 6t - 4$	M1 A1
$6t - 4 = 0 \ \Rightarrow t = \frac{2}{3}$	M1 A1
$s = \int 3t^2 - 4t + 3\,dt = t^3 - 2t^2 + 3t\ (+c)$	M1 A1
$t = \frac{2}{3} \Rightarrow s = -\frac{16}{27} + 2$ so distance is $\frac{38}{27}$ m	M1 A1

[8]

3. (a)

$\frac{dv}{dt} = 8 - 2t$	M1	
$8 - 2t = 0$	M1	
Max $v = 8 \times 4 - 4^2 = 16$ (ms^{-1})	M1A1	4

(b)

$\int 8t - t^2 dt = 4t^2 - \frac{1}{3}t^3 (+C)$	M1A1
($t = 0$, displacement = 0 $\Rightarrow$ c=0)	
$4T^2 - \frac{1}{3}T^3 = 0$	DM1
$T^2\left(4 - \frac{T}{3}\right) = 0 \Rightarrow T = 0{,}12$	DM1

$T = 12$ (seconds) A1 5

[9]

4. (a) $v = 10t - 2t^2$, s = $\int v dt$ M1

$= 5t^2 - \frac{2t^3}{3} (+C)$ A1

$t = 6 \Rightarrow s = 180 - 144 = \underline{36}$ (m) A1 3

(b) $\underline{s} = \int v dt = \underline{\frac{-432t^{-1}}{-1}}(+k) = \underline{\frac{432}{t}}(+k)$ $\underline{\text{B1}}$

t = 6, s = "36" $\Rightarrow 36 = \frac{432}{6} + K$ M1 *

$\Rightarrow K = -36$ A1

At $t = 10$, $s = \frac{432}{10} - 36 = \underline{7.2}$(m) d * M1

$\underline{\text{A1}}$ 5

[8]

5 (a) $0 \le t \le 4$: $a = 8 - 3t$ M1

$a = 0 \Rightarrow t = 8/3$ s DM1

$\rightarrow v = 8.\frac{8}{3} - \frac{3}{2}.\left(\frac{8}{3}\right)^2 = \frac{32}{3}$ (m/s) DM1A1 4

second M1 dependent on the first, and third dependent on the second.

(b) $s = 4t^2 - r^3/2$ M1

$t = 4$: $s = 64 - 64/2 = \underline{32\text{ m}}$ M1A1 3

(c) $t > 4$: $v = 0 \Rightarrow t = \underline{8\text{ s}}$ B1 1

(d) *Either*

$t > 4$ $s = 16t - t^2$ $(+C)$ M1

$t = 4$, $s = 32 \rightarrow C = -16 \Rightarrow s = 16t - t^2 - 16$ M1A1

$t = 10 \rightarrow s = 44$ m M1A1

But direction changed, so: $t = 8$, $s = 48$ M1

Hence total dist travelled = 48 + 4 = $\underline{52\text{ m}}$ DM1A1 8

Or (probably accompanied by a sketch?)

$t = 4$ $v = 8$, $t = 8$ $v = 0$, so area under line = $\frac{1}{2} \times (8 - 4) \times 8$ M1A1A1

$t = 8 \quad v = 0$, $t = 10 \quad v = –4$, so area above line = $\frac{1}{2} \times (10 - 8) \times 4$ M1A1A1

$\therefore$ total distance = 32 (from B) + 16 + 4 = <u>52 m</u> M1A1 8

Or M1, A1 for $t > 4$ $\frac{dv}{dt} = -2$, = constant

$t = 4$, $v = 8$; $t = 8$, $v = 0$; $t = 10$, $v = –4$

M1, A1 $s = \frac{u+v}{2}t = \frac{32}{2}t, = 16$ working for $t = 4$ to $t = 8$

M1, A1 $s = \frac{u+v}{2}t = \frac{-4}{2}t, = 4$ working for $t = 8$ to $t = 10$

M1, A1 total = 32 + 14 + 4, = 52

M1 Differentiate to obtain acceleration

DM1 set acceleration = 0 and solve for t

DM1 use their t to find the value of v

A1 32/3, 10.7 or better

OR using trial and improvement:

M1 Iterative method that goes beyond integer values

M1 Establish maximum occurs for t in an interval no bigger than $2.5 < t < 3.5$

M1 Establish maximum occurs for t in an interval no bigger than $2.6 < t < 2.8$

OR M1 Find/state the coordinates of both points where the curve cuts the x axis.

DM1 Find the midpoint of these two values

M1A1 as above.

OR M1 Convincing attempt to complete the square:

DM1 substantially correct $8t - \frac{3t^2}{2} = -\frac{3}{2}(t - \frac{8}{3})^2 + \frac{3}{2} \times \frac{64}{9}$

DM1 Max value = constant term

A1 CSO

M1 Integrate the correct expression

DM1 Substitute $t = 4$ to find distance ($s = 0$ when $t = 0$ – condone omission/ignoring of constant of integration)

A1 32(m) only

B1 $t = 8$ (s) only

M1 Integrate $16 – 2t$

M1 Use $t = 4$, s = their value from (b) to find the value of the constant of integration.
or 32 + integral with a lower limit of 4 (in which case you probably see these two marks occurring with the next two. First A1 will be for 4 correctly substituted.)

A1 $s = 16t – t^2 – 16$ or equivalent

M1 substitute $t = 10$

A1 44

M1 Substitute $t = 8$ (their value from (c))

DM1 Calculate total distance (M mark dependent on the previous M mark.)

A1 52 (m)

OR the candidate who recognizes $v = 16 – 2t$ as a straight line can divide the shape into two triangles:
- M1 distance for $t = 4$ to t = candidate's 8 = ½ × change in time × change in speed.
- A1 8 – 4
- A1 8 – 0
- M1 distance for t = their 8 to $t = 10$ = ½ × change in time × change in speed.
- A1 10 – 8
- A1 0 – (–4)
- M1 Total distance = their (b) plus the two triangles (=32 + 16 + 4)
- A1 52(m)

NB: This order on epen grid (the A's and M's will not match up.)

[16]

6. $a = 5 – 2t \Rightarrow v = 5t – t^2, + 6$ M1 A1 A1

$v = 0 \Rightarrow t^2 –5t – 6 = 0$ indep M1

$(t – 6)(t + 1) = 0$ dep M1

$t = 6$ s A1

[6]

7. (a) $I = \pm 0.5(16\mathbf{i} + 20\mathbf{j} – (–30\mathbf{i}))$ M1

$= \pm(23\mathbf{i} + 10\text{j})$ Indep M1

magn = $\sqrt{(23^2 + 10^2)} \approx$ <u>25.1 Ns</u> Indep M1 A1 4

	(b)	$v = 16\mathbf{i} + (20 - 10t)\mathbf{j}$	M1	
		$t = 3 \Rightarrow \mathbf{v} = 16\mathbf{i} - 10\mathbf{j}$	indep M1	
		$v = \sqrt{(16^2 + 10^2)} \approx 18.9 \text{ ms}^{-1}$	indep M1 A1	4
				[8]

8.	(a)	$\dot{\mathbf{r}} = (2t + 4)\mathbf{i} + (3 - 3t^2)\mathbf{j}$		M1 A1	
		$\dot{\mathbf{r}}_3 = 10\mathbf{i} - 24\mathbf{j}$	substituting $t = 3$	M1	
		$\lvert \dot{\mathbf{r}}_3 \rvert = \sqrt{(10^2 + 24^2)}) = 26 \text{ (m s}^{-1})$		M1 A1	5
	(b)	$0.4 (\mathbf{v} - (10\mathbf{i} - 0\ 24\mathbf{j}) = 8\mathbf{i} - 12\mathbf{j}$	ft their $\dot{\mathbf{r}}_3$	M1 A1ft	
		$\mathbf{v} = 30\ \mathbf{I} - 54\mathbf{j}$ (m s^{-1})		A1	3
					[8]

9.	(a)	$\mathbf{v} = (18 - 12t^2)\mathbf{I} + 2ct\mathbf{j}$	M1 A1 A1	
		$t = \frac{3}{2}$: $\mathbf{v} = -9\mathbf{i} + 3c\mathbf{j}$	M1	
		$\lvert \mathbf{v} \rvert = 15 \Rightarrow 9^2 + (3c)^2 = 15^2$	M1	
		$\Rightarrow (3c)^2 = 144 \Rightarrow c = 4$	A1	6
	(b)	$\mathbf{a} = -24t\mathbf{i} + 8\mathbf{j}$	M1	
		$t = \frac{3}{2}$ $\mathbf{a} = -36\mathbf{i} + 8\mathbf{j}$	M1	
			A1ft	3
				[9]

10.	(a)	$\mathbf{p} = (2t^2 - 7t)\mathbf{I} - 5t\mathbf{j}, + 3\mathbf{i} + 5\mathbf{j}$		M1, M1	
		$= (2t^2 - 7t + 3)\mathbf{I} + (5 - 5t)\mathbf{j}$		A1+A1	4
	(b)	$\mathbf{q} = (2\mathbf{i} - 3\mathbf{j})t - 7\mathbf{i}$		M1 A1	
		$\mathbf{j}$: $5 - 5t = -3t \Rightarrow t = 2.5$	equating and solving	M1 A1	
		At $t = 2.5$ $\mathbf{i}$: $p_x = 2 \times 2.5^2 - 7 \times 2.5 + 3 = -2$			
		$q_x = 2 \times 2.5 - 7 = -2$	both	M1	
		$p_x = q_x \Rightarrow$ collision		cso A1	6
					[10]

Alternative in (b)

i : $2t^2 - 7t + 3 = 2t - 7 \Rightarrow 2t^2 - 9t + 10 = 0$
$t = 2, 2.5$ equating and solving M1 A1

At $t = 2.5$ **j** : $p_y = 5 - 5 \times 2.5 = -7.5$
$q_y = -3 \times 2.5 = -7.5$ both M1
$p_y = q_y \Rightarrow$ collision cso A1

In alternative, ignore any working associated with $t = 2$

11. $x = \int 6t - 2t^2 \, dt$ M1
$= 3t^2 - \frac{2}{3}t^3 \ (+ C)$ A1
$v = 0 \Rightarrow 6t - 2t^2 = 0 \Rightarrow t = 3$ (or 0) M1
$t = 3$: $x = (3 \times 9) - (\frac{2}{3} \times 27) = 9$ m M1 A1

[5]

12. (a) $v = \int a \, dt = 2t^2 - 8t \ (+c)$ M1 A1
Using $v = 6$, $t = 0$; $v = 2t^2 - 8t + 6$ M1 A1 4

(b) $v = 0 \Rightarrow 2t^2 - 8t + 6 = 0, \Rightarrow t = 1,3$ M1 A1
$S = \int (2t^2 - 8t + 6)\mathrm{d}t = \left[\frac{2}{3}t^3 - 4t^2 + 6t\right]$ M1 A2, 1, 0
$= 0 - 2\frac{2}{3}$ M1
Distance is $(\pm)2\frac{2}{3}$ m A1 7

[11]

1. This question proved very accessible and gave most candidates a confident start to the paper. There were very few incorrect answers, with the overwhelming majority integrating the given acceleration correctly. Any errors in the integration were mostly when $3t^2$ was not divided by 2. There was some confusion about the constant of integration in a few cases, often taken in error to be zero. Nearly all candidates set their velocity expressions equal to 6 and attempted to solve the resulting quadratic equation. There were some basic algebraic or arithmetical slips resulting in incorrect equations. A method was not always shown in the solution of a quadratic. This should be discouraged as credit can be given for correct working if it is seen.

There were a small number of candidates who tried to apply "*suvat*" to the motion, losing 5 out of the 6 marks available.

2. The majority of candidates offered confident responses to this opening question. Most of them successfully integrated the given velocity to find out the displacement of the particle. When it came to finding out the time of minimum velocity, most candidates used calculus again to find acceleration and made it equal to zero but some preferred to complete the square or use the expression for the turning point of a parabola. A few candidates attempted to find the time when the velocity of the particle was zero, believing this to be the minimum. A common error was find the minimum velocity and substitute this, rather than the time, into the displacement equation.

3. This question provided the opportunity for candidates to show that they could both differentiate the velocity function to find the acceleration and integrate it to find the displacement. In general both were done successfully, although as usual there were candidates who incorrectly attempted to solve the problem using constant acceleration formulae.

Although the majority of candidates used differentiation in part (a), there was also a large number who treated it by completing the square, and they were often successful in this approach. A number of candidates produced a table of discrete time values and corresponding speeds of the particle. Unfortunately they rarely scored full marks for their effort as the supporting statement about the symmetry of a quadratic function was usually missing. The most common error among candidates using differentiation was to stop when they had found the time and not go on to find the speed. In part (b) it would have been reassuring to have seen more candidates - even the successful ones - giving a more rigorous treatment of the constant of integration. Algebraic errors in solving the equation $4T^2 - \frac{1}{3}T^3 = 0$ were surprisingly common.

3. A few candidates were clearly confused by velocity being defined in terms of two separate functions. Nevertheless, virtually all candidates knew they had to integrate the relevant expression for velocity in order to find the displacement and they did this correctly in part (a). As the constant was zero in this part of the question, candidates who had overlooked it were not penalised. There were occasional mistakes such as differentiating instead of integrating, and some candidates who tried to use the equations for constant acceleration.

In part (b), although most correctly integrated the expression, for those that went along the indefinite integral route, the constant of integration was often just assumed to be zero because the displacement was zero at the start. Several candidates even demonstrated that the constant of integration was zero, apparently having no problem with equating 432/0 to zero! These candidates clearly did not realise that the expression was not relevant at the start. Those who found the definite integral were generally more successful. Other errors in part (b) included using $t = 4$, using $t = 7$ as a lower limit for the second integral (apparently not recognising the continuous nature of time), or reaching the correct solution but then adding the answer from (a) a second time.

5. Completely correct solutions to this question were rare, with parts (b) and (c) proving to be a better source of marks than parts (a) or (d).

(a) There are several possible methods for finding the maximum speed in this interval. The expected method was to differentiate, find the value of t for which the acceleration is equal to zero, and use this to find the corresponding value of v. Candidates using this approach sometimes got as far as the value for t and then stopped as if they thought they had answered the question. As an alternative, candidates who recognised this as part of a parabola, either went on to complete the square (with considerable success despite the nature of the algebra involved), or found the average of the two times when the speed is zero to locate the time for maximum speed and hence the speed, or simply quoted formulae for the location of the turning point. Many candidates simply substituted integer values of t in to the formula for v and stated their largest answer. This alone was not sufficient. Although it is possible to arrive at the correct answer using trial and improvement, most candidates who embarked on this route failed to demonstrated that their answer was indeed a maximum – they usually offered a sequence of increasing values, but did not demonstrate that they had located the turning point in an interval of appropriate width.

(b) Many candidates answered this correctly – even those who did not differentiate in part (a) did choose to integrate here. There is a false method, assuming constant speed throughout the interval, which gives the answer 32 incorrectly by finding the speed when $t = 4$ and multiplying the result by 4 – many candidates used this without considering the possibility of variable speed and acceleration.

(c) This was usually answered correctly, but some candidates appeared to think that they were being asked to find out when $8t - \frac{3}{2}t^2 = 0$ or when $8t - \frac{3}{2}t^2 = 16 - 2t$.

(d) Those candidates who realised that the particle was now moving with uniform acceleration had the simple task of finding the area of two triangles, assuming that they appreciated the significance of $v < 0$ for $t > 8$. Alternatively they could use the equations for motion under uniform acceleration, with the same proviso. For the great majority of candidates, this was about integration and choosing appropriate limits. The integration itself was usually correct, but common errors included ignoring the lower limit of the interval, or not using $s = 32$ when $t = 4$, and stopping after using the upper limit of $t = 10$. Some candidates thought that the limits for t should be from $t = 0$ to $t = 6$, and a large number thought that they should be starting from $t = 5$. Very few of the candidates who found the integral went on to consider what happened between $t = 8$ and $t = 10$.

6. This proved to be an easy starter and was generally very well answered with the vast majority of candidates scoring 5 or 6 marks. There were some errors in integration, with some candidates failing to include a constant and some unable to solve the required quadratic equation. Of those that could, some failed to reject the negative solution. A few candidates assumed constant acceleration and scored little.

7. There were few errors on this question Only a few candidates failed to use vectors to calculate the impulse in part (a) but some forgot to calculate the magnitude of their vector. The second part was mostly completely correct. Candidates need to read the question carefully and ensure that they answer the question asked. Those who forgot to calculate magnitudes lost four marks in this question!

8. Most candidates realised that they needed to differentiate, although there was the odd integration. Having found a velocity vector some then failed to find the modulus to obtain the speed. In part (b), a few used $\mathbf{I} = m(\mathbf{u} - \mathbf{v})$ and a very small number worked with scalars, but generally candidates reached the correct vector solution. Some candidates thought they then had to find the magnitude of their vector – they were not penalised for this.

9. There were many fully correct solutions but there was some sloppy use of vector notation and also evidence of poor algebra. Most knew that they needed to differentiate but some lost the **i**'s and **j**'s. Others, in (a), were unable to deal with the magnitude correctly and simply put 15 = -9 + c. In the second part many unnecessarily went on to find the magnitude of the acceleration.

10. Most recognised that integration was needed in part (a), although a few used an inappropriate formula such as $\mathbf{r} = \mathbf{r}_0 + \mathbf{V}t$. Almost all candidates knew how to incorporate the initial position but errors in manipulation were seen and an error in bracketing frequently led to the incorrect $\left(2t^2 - 7t + 3\right)\mathbf{i} - \left(5 + 5t\right)\mathbf{j}$. This lost the last mark in (a) and from this result it was impossible to complete part (b) correctly. However nearly all candidates were able to demonstrate the method needed in (b) and the question was a substantial source of marks for the great majority of candidates.

11. This proved to be an easy starter and there were very few errors. A few differentiated instead of integrating and there was the odd algebraic or arithmetical slip.

12. Full marks were common for this question. A few candidates omitted the constant from the first integration or tried to use $v = u + at$ to find the velocity. Candidates needed to show, in part (a), how the +6 was obtained in the velocity equation and, in part (b), it was necessary to be clear that the distance travelled *between* the two times found had been calculated. In part (b), it was encouraging to note how many candidates, on obtaining an answer of $-\frac{2}{3}$, re-wrote the answer without the negative sign and with a unit, making some suitable comment on the nature of "distance".