

## Stationary Points

Exercise 1ZF Q3

Find tgt at (2,5)

$$\text{when } x=2 \quad \frac{dy}{dx} = 2(2) = 4$$

$$y - y_1 = m(x - x_1)$$

$$y - 5 = 4(x - 2)$$

$$y - 5 = 4x - 8$$

$$\underline{\underline{y = 4x - 3}}$$

$$y = x^2 + 1$$

$$\frac{dy}{dx} = 2x$$

Find normal at (1,2)

$$\text{when } x=1, \quad \frac{dy}{dx} = 2(1) = 2$$

$$\text{gradient of normal} = -\frac{1}{2}$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{1}{2}(x - 1)$$

$$y - 2 = -\frac{1}{2}x + \frac{1}{2}$$

$$\underline{\underline{y = -\frac{1}{2}x + \frac{5}{2}}}$$

Sub for y

$$-\frac{1}{2}x + \frac{5}{2} = 4x - 3$$

$$-x + 5 = 8x - 6$$

$$+5+6 = 8x+x$$

$$11 = 9x$$

$$x = \frac{11}{9}$$

$$y = 4\left(\frac{11}{9}\right) - 3$$

$$y = \frac{44}{9} - \frac{27}{9}$$

$$y = \frac{17}{9}$$

## Increasing and Decreasing Functions

$f(x)$  is increasing on the interval  $[a,b]$

if  $f'(x) \geq 0$  for all  $x \in (a,b)$

Decreasing if  $f'(x) \leq 0$

## Examples

$$1) f(x) = 3x^2 + 8x + 2$$

$$f'(x) = 6x + 8$$

increasing function when  $6x + 8 \geq 0$

$$6x \geq -8$$

$$x \geq -\frac{4}{3}$$

$$x \geq -\frac{4}{3}$$

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$$2) f(x) = x^2 - 9x$$

$$f'(x) = 2x - 9$$

decreasing function when  $2x - 9 \leq 0$

$$2x \leq 9$$

$$x \leq \frac{9}{2}$$

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$$3) f(x) = 4 - x(2x^2 + 3)$$

$$f(x) = 4 - 2x^3 - 3x$$

$$f'(x) = -6x^2 - 3$$

$$= -3(2x^2 + 1)$$

$\leq 0$  for all  $x$

since  $2x^2 + 1 > 0$

$$\text{and } -3 < 0$$

thus  $x - ve \Rightarrow -ve$

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Stationary Points

### Maxima, Minima , Points of Inflection

Example  $y = 2x^3 - 15x^2 + 24x + 6$

$$\frac{dy}{dx} = 6x^2 - 30x + 24$$

At st pt  $\frac{dy}{dx} = 0$

$$0 = 6x^2 - 30x + 24$$

$$0 = x^2 - 5x + 4$$

$$0 = (x - 4)(x - 1)$$

$$\Rightarrow x = 4 \text{ or } x = 1$$

when  $x = 4$   $y = 2(4)^3 - 15(4)^2 + 24(4) + 6 = -10$

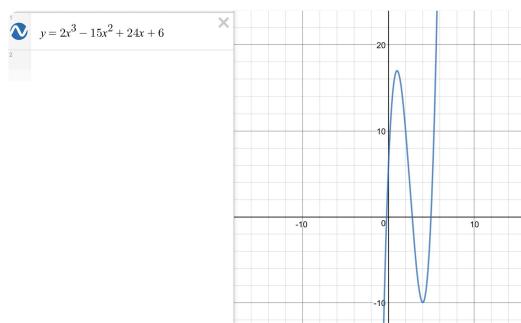
$x = 1$   $y = 2(1)^3 - 15(1)^2 + 24(1) + 6 = 17$

st pts at  $(1, 17)$  and  $(4, -10)$

$$\frac{d^2y}{dx^2} = 12x - 30$$

when  $x = 1$   $\frac{d^2y}{dx^2} = 12(1) - 30 = -18 < 0$   
 $\therefore$  max at  $(1, 17)$

when  $x = 4$   $\frac{d^2y}{dx^2} = 12(4) - 30 = 18 > 0$   
 $\therefore$  min at  $(4, -10)$



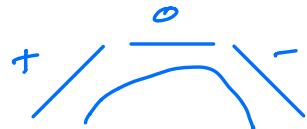
## Alternative Method of Determining Nature of Stationary Points

st pt when  $x = 1$

$$\frac{dy}{dx} = 6x^2 - 30x + 24$$

when  $x = 0.9$ ,  $\frac{dy}{dx} = 6(0.9)^2 - 30(0.9) + 24 = 1.86 \text{ +ve}$

when  $x = 1.1$ ,  $\frac{dy}{dx} = 6(1.1)^2 - 30(1.1) + 24 = -1.74 \text{ -ve}$

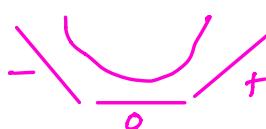


$\therefore$  a maximum at  $(1, 17)$

st pt when  $x = 4$

when  $x = 3.9$ ,  $\frac{dy}{dx} = 6(3.9)^2 - 30(3.9) + 24 = -1.74 \text{ -ve}$

when  $x = 4.1$ ,  $\frac{dy}{dx} = 6(4.1)^2 - 30(4.1) + 24 = 1.86 \text{ +ve}$



$\therefore$  a minimum at  $(4, -10)$