Exercise 12C Differentiation

$$\frac{d}{dx} ax^n = nax^{n-1}$$

1f)

$$f(x) = \sqrt[3]{x}$$
$$f(x) = x^{\frac{1}{3}}$$
$$f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$$

1e)

$$f(x) = \frac{1}{\sqrt[3]{x}}$$
$$f(x) = x^{-\frac{1}{3}}$$
$$f'(x) = -\frac{1}{3}x^{-\frac{4}{3}}$$

1r)

$$f(x) = \frac{x^6}{x^3}$$
$$f(x) = x^3$$
$$f'(x) = 3x^2$$

2e)

$$y = 6x^{5/4}$$
$$\frac{dy}{dx} = \frac{5}{4} \times 6x^{\frac{1}{4}}$$
$$\frac{dy}{dx} = \frac{15}{2}x^{\frac{1}{4}}$$

2j)

$$y = \sqrt{\frac{5x^4 \times 10x}{2x^2}}$$

$$y = \sqrt{\frac{50x^5}{2x^2}}$$

$$y = \sqrt{25x^3}$$

$$y = 5x^{3/2}$$

$$\frac{dy}{dx} = \frac{3}{2} \times 5x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{15}{2}x^{\frac{1}{2}}$$

Exercise 1e, k, q 2d, i

1e) $f(x) = x^{\frac{1}{4}} \Rightarrow f'(x) = \frac{1}{4}x^{-3/4}$

1k) $f(x) = \frac{1}{\sqrt{x}}$

$f(x) = x^{-\frac{1}{2}} \Rightarrow f'(x) = -\frac{1}{2}x^{-3/2}$

1q) $f(x) = \frac{x^3}{x^2}$

$f(x) = x \Rightarrow f'(x) = 1$

2d) $y = 20x^{\frac{1}{4}} \Rightarrow \frac{dy}{dx} = 5x^{-3/4}$

2i) $y = -\frac{2}{\sqrt{x}}$ $y = -2x^{-\frac{1}{2}} \Rightarrow \frac{dy}{dx} = x^{-3/2}$

Tangents and Normals

Example $y = x^3 - 7x + 2$

Point on curve $P(2, -4)$

Find the equations of the tangent and normal to the curve at this point

$$y = x^3 - 7x + 2$$

$$\frac{dy}{dx} = 3x^2 - 7$$

when $x = 2$, $\frac{dy}{dx} = 3(2)^2 - 7 = 5$

$\therefore$ gradient $= 5$

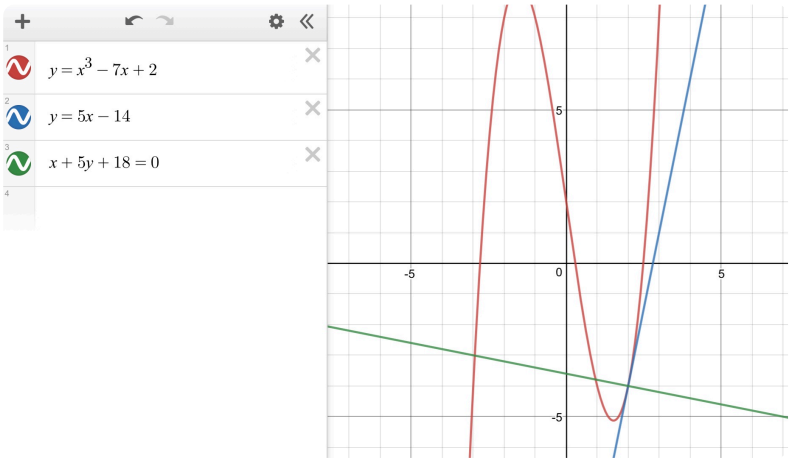
Tangent at P

$$y - y_1 = m(x - x_1)$$
$$y - -4 = 5(x - 2)$$
$$y + 4 = 5x - 10$$
$$\underline{y = 5x - 14}$$

Normal at P has gradient $= -\frac{1}{5}$

$$y - y_1 = m(x - x_1)$$
$$y - -4 = -\frac{1}{5}(x - 2)$$
$$y + 4 = -\frac{1}{5}x + \frac{2}{5}$$
$$y = -\frac{1}{5}x - \frac{18}{5}$$
$$5y = -x - 18$$

$$x + 5y + 18 = 0$$



Exercise 12F

1f) $y = x^2 - \frac{7}{x^2}$ $\quad P(1, -6)$

$y = x^2 - 7x^{-2}$

$\frac{dy}{dx} = 2x + 14x^{-3}$

$\frac{dy}{dx} = 2x + \frac{14}{x^3}$

when $x = 1$, $\frac{dy}{dx} = 2(1) + \frac{14}{1^3} = 16$

$y - y_1 = m(x - x_1)$

$y - -6 = 16(x - 1)$

$y + 6 = 16x - 16$

$y = 16x - 22$

2 b) $y = x^2 - \frac{8}{\sqrt{x}}$ $P(4, 12)$

$y = x^2 - 8x^{-\frac{1}{2}}$

$\frac{dy}{dx} = 2x + 4x^{-3/2}$

$\frac{dy}{dx} = 2x + \frac{4}{x^{3/2}}$

when $x = 4$ $\frac{dy}{dx} = 2(4) + \frac{4}{(4)^{3/2}}$

$\frac{dy}{dx} = 8 + \frac{1}{2} = \frac{17}{2}$

Gradient of Normal $= -\frac{2}{17}$

$y - y_1 = m(x - x_1)$

$y - 12 = -\frac{2}{17}(x - 4)$

$17y - 204 = -2x + 8$

$2x + 17y - 212 = 0$

Homework Exercise 12F