Section B (36 marks)

8 The upper and lower surfaces of a coal seam are modelled as planes ABC and DEF, as shown in Fig. 8. All dimensions are metres.

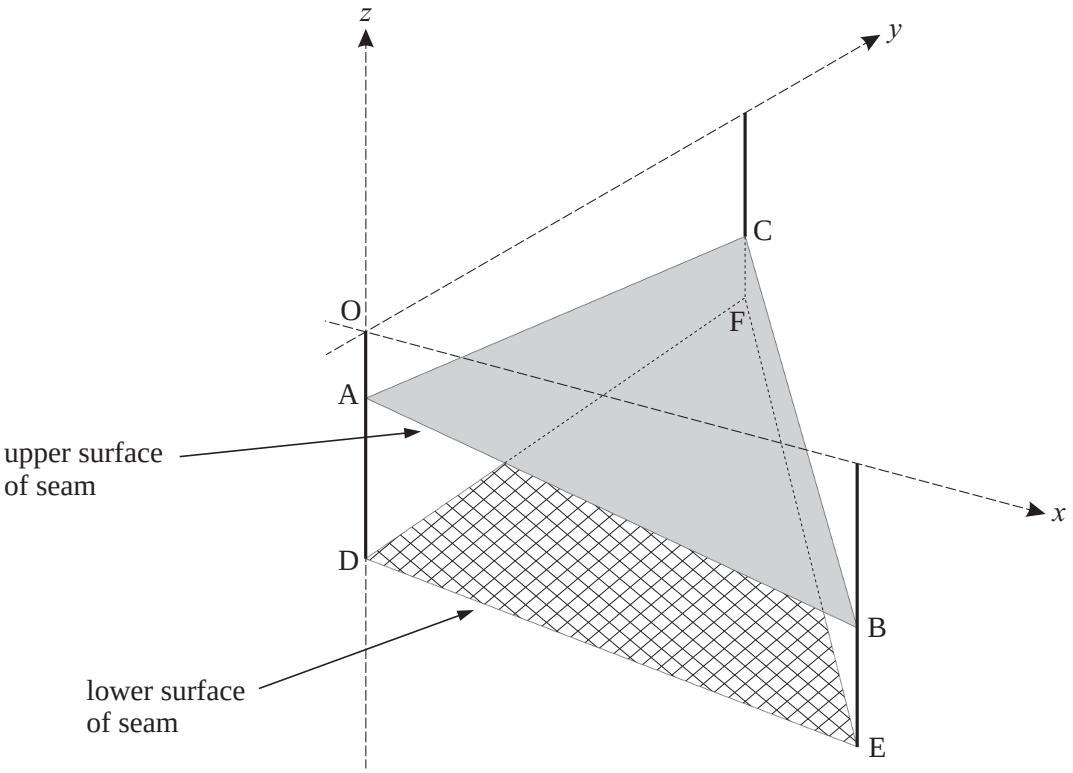

Fig. 8

Relative to axes O*x* (due east), O*y* (due north) and O*z* (vertically upwards), the coordinates of the points are as follows.

A: (0, 0, −15) B: (100, 0, −30) C: (0, 100, −25)
D: (0, 0, −40) E: (100, 0, −50) F: (0, 100, −35)

(i) Verify that the cartesian equation of the plane ABC is $3x + 2y + 20z + 300 = 0$. [3]

(ii) Find the vectors $\overrightarrow{DE}$ and $\overrightarrow{DF}$. Show that the vector $2\mathbf{i} - \mathbf{j} + 20\mathbf{k}$ is perpendicular to each of these vectors. Hence find the cartesian equation of the plane DEF. [6]

(iii) By calculating the angle between their normal vectors, find the angle between the planes ABC and DEF. [4]

It is decided to drill down to the seam from a point R (15, 34, 0) in a line perpendicular to the upper surface of the seam. This line meets the plane ABC at the point S.

(iv) Write down a vector equation of the line RS.

Calculate the coordinates of S. [5]

9 A skydiver drops from a helicopter. Before she opens her parachute, her speed $v\,\mathrm{m\,s^{-1}}$ after time t seconds is modelled by the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = 10\mathrm{e}^{-\frac{1}{2}t}.$$

When $t = 0$, $v = 0$.

(i) Find v in terms of t. [4]

(ii) According to this model, what is the speed of the skydiver in the long term? [2]

She opens her parachute when her speed is $10\,\mathrm{m\,s^{-1}}$. Her speed t seconds after this is $w\,\mathrm{m\,s^{-1}}$, and is modelled by the differential equation

$$\frac{\mathrm{d}w}{\mathrm{d}t} = -\tfrac{1}{2}(w-4)(w+5).$$

(iii) Express $\dfrac{1}{(w-4)(w+5)}$ in partial fractions. [4]

(iv) Using this result, show that $\dfrac{w-4}{w+5} = 0.4\mathrm{e}^{-4.5t}$. [6]

(v) According to this model, what is the speed of the skydiver in the long term? [2]