PMT

Section B (36 marks)

8 The upper and lower surfaces of a coal seam are modelled as planes ABC and DEF, as shown in Fig. 8. All dimensions are metres.



Fig. 8

Relative to axes Ox (due east), Oy (due north) and Oz (vertically upwards), the coordinates of the points are as follows.

A: (0, 0, -15) B: (100, 0, -30) C: (0, 100, -25) D: (0, 0, -40) E: (100, 0, -50) F: (0, 100, -35)

- (i) Verify that the cartesian equation of the plane ABC is 3x + 2y + 20z + 300 = 0. [3]
- (ii) Find the vectors \overrightarrow{DE} and \overrightarrow{DF} . Show that the vector $2\mathbf{i} \mathbf{j} + 20\mathbf{k}$ is perpendicular to each of these vectors. Hence find the cartesian equation of the plane DEF. [6]
- (iii) By calculating the angle between their normal vectors, find the angle between the planes ABC and DEF. [4]

It is decided to drill down to the seam from a point R (15, 34, 0) in a line perpendicular to the upper surface of the seam. This line meets the plane ABC at the point S.

(iv) Write down a vector equation of the line RS.

Calculate the coordinates of S.

[5]

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[4]

[4]

9

A skydiver drops from a helicopter. Before she opens her parachute, her speed $v \,\mathrm{m \, s^{-1}}$ after time *t* seconds is modelled by the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = 10\mathrm{e}^{-\frac{1}{2}t}$$

When t = 0, v = 0.

- (i) Find v in terms of t.
- (ii) According to this model, what is the speed of the skydiver in the long term? [2]

She opens her parachute when her speed is 10 m s^{-1} . Her speed *t* seconds after this is $w \text{ m s}^{-1}$, and is modelled by the differential equation

$$\frac{\mathrm{d}w}{\mathrm{d}t} = -\frac{1}{2}(w-4)(w+5)$$

(iii) Express $\frac{1}{(w-4)(w+5)}$ in partial fractions.

(iv) Using this result, show that
$$\frac{w-4}{w+5} = 0.4e^{-4.5t}$$
. [6]

(v) According to this model, what is the speed of the skydiver in the long term? [2]

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