Section B (36 marks)

6 Fig. 6 shows the arch ABCD of a bridge.

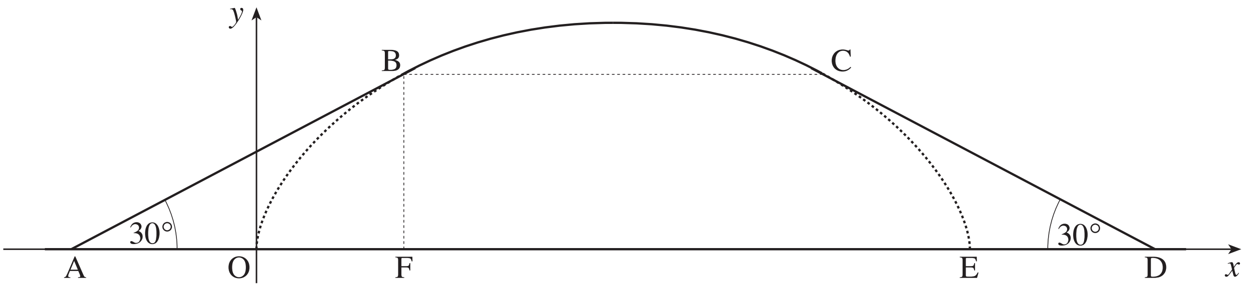

Fig. 6

The section from B to C is part of the curve OBCE with parametric equations

$$x = a(\theta - \sin\theta),\ y = a(1 - \cos\theta) \text{ for } 0 \leqslant \theta \leqslant 2\pi,$$

where a is a constant.

(i) Find, in terms of a,

(A) the length of the straight line OE,

(B) the maximum height of the arch. [4]

(ii) Find $\dfrac{dy}{dx}$ in terms of θ. [3]

The straight line sections AB and CD are inclined at 30° to the horizontal, and are tangents to the curve at B and C respectively. BC is parallel to the x-axis. BF is parallel to the y-axis.

(iii) Show that at the point B the parameter θ satisfies the equation

$$\sin\theta = \frac{1}{\sqrt{3}}(1 - \cos\theta).$$

Verify that $\theta = \frac{2}{3}\pi$ is a solution of this equation.

Hence show that BF $= \frac{3}{2}a$, and find OF in terms of a, giving your answer exactly. [6]

(iv) Find BC and AF in terms of a.

Given that the straight line distance AD is 20 metres, calculate the value of a. [5]

7

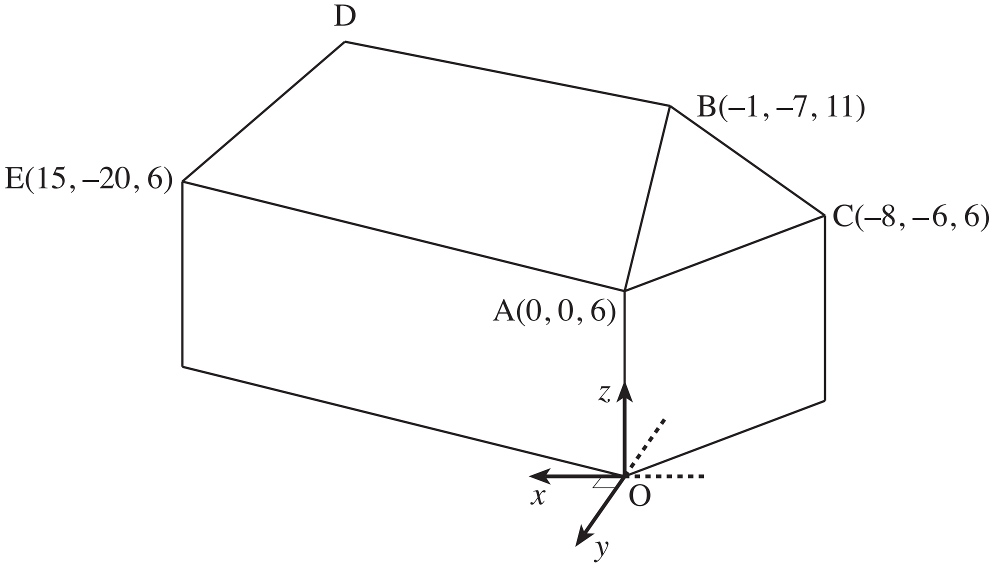

Fig. 7

Fig. 7 illustrates a house. All units are in metres. The coordinates of A, B, C and E are as shown. BD is horizontal and parallel to AE.

(i) Find the length AE. [2]

(ii) Find a vector equation of the line BD. Given that the length of BD is 15 metres, find the coordinates of D. [4]

(iii) Verify that the equation of the plane ABC is

$$-3x + 4y + 5z = 30.$$

Write down a vector normal to this plane. [4]

(iv) Show that the vector $\begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix}$ is normal to the plane ABDE. Hence find the equation of the plane ABDE. [4]

(v) Find the angle between the planes ABC and ABDE. [4]