

## Section B (36 marks)

- 6 In Fig. 6, OAB is a thin bent rod, with  $OA = a$  metres,  $AB = b$  metres and angle  $OAB = 120^\circ$ . The bent rod lies in a vertical plane. OA makes an angle  $\theta$  above the horizontal. The vertical height BD of B above O is  $h$  metres. The horizontal through A meets BD at C and the vertical through A meets OD at E.

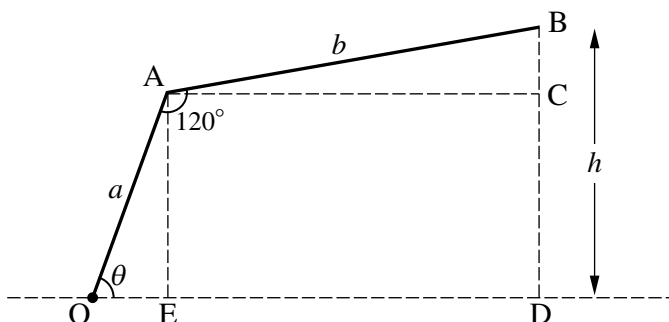


Fig. 6

- (i) Find angle BAC in terms of  $\theta$ . Hence show that

$$h = a \sin \theta + b \sin(\theta - 60^\circ). \quad [3]$$

- (ii) Hence show that  $h = (a + \frac{1}{2}b) \sin \theta - \frac{\sqrt{3}}{2}b \cos \theta$ . [3]

The rod now rotates about O, so that  $\theta$  varies. You may assume that the formulae for  $h$  in parts (i) and (ii) remain valid.

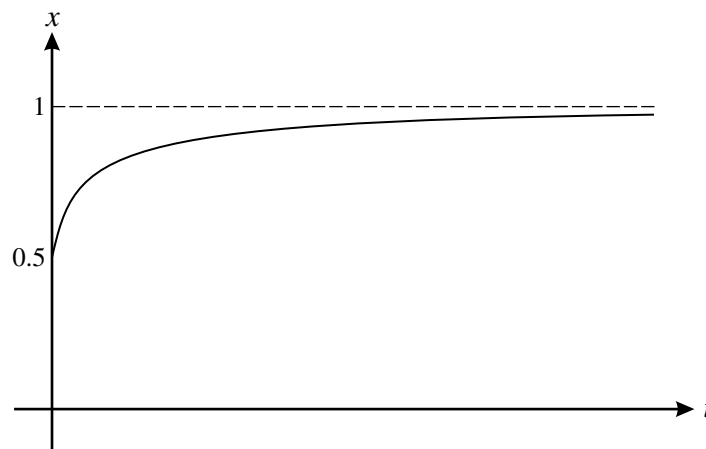
- (iii) Show that OB is horizontal when  $\tan \theta = \frac{\sqrt{3}b}{2a+b}$ . [3]

In the case when  $a = 1$  and  $b = 2$ ,  $h = 2 \sin \theta - \sqrt{3} \cos \theta$ .

- (iv) Express  $2 \sin \theta - \sqrt{3} \cos \theta$  in the form  $R \sin(\theta - \alpha)$ . Hence, for this case, write down the maximum value of  $h$  and the corresponding value of  $\theta$ . [7]

[Question 7 is printed overleaf.]

- 7 Fig. 7 illustrates the growth of a population with time. The proportion of the ultimate (long term) population is denoted by  $x$ , and the time in years by  $t$ . When  $t = 0$ ,  $x = 0.5$ , and as  $t$  increases,  $x$  approaches 1.



**Fig. 7**

One model for this situation is given by the differential equation

$$\frac{dx}{dt} = x(1 - x).$$

- (i) Verify that  $x = \frac{1}{1 + e^{-t}}$  satisfies this differential equation, including the initial condition. [6]
- (ii) Find how long it will take, according to this model, for the population to reach three-quarters of its ultimate value. [3]

An alternative model for this situation is given by the differential equation

$$\frac{dx}{dt} = x^2(1 - x),$$

with  $x = 0.5$  when  $t = 0$  as before.

- (iii) Find constants  $A$ ,  $B$  and  $C$  such that  $\frac{1}{x^2(1 - x)} = \frac{A}{x^2} + \frac{B}{x} + \frac{C}{1 - x}$ . [4]
- (iv) Hence show that  $t = 2 + \ln\left(\frac{x}{1 - x}\right) - \frac{1}{x}$ . [5]
- (v) Find how long it will take, according to this model, for the population to reach three-quarters of its ultimate value. [2]

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