PMT

[2]

Section B (36 marks)

- 7 Scientists can estimate the time elapsed since an animal died by measuring its body temperature.
 - (i) Assuming the temperature goes down at a constant rate of 1.5 degrees Fahrenheit per hour, estimate how long it will take for the temperature to drop
 - (A) from 98 $^{\circ}$ F to 89 $^{\circ}$ F,
 - (*B*) from 98 $^{\circ}$ F to 80 $^{\circ}$ F.

In practice, rate of temperature loss is not likely to be constant. A better model is provided by Newton's law of cooling, which states that the temperature θ in degrees Fahrenheit *t* hours after death is given by the differential equation

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = -k(\theta - \theta_0),$$

where $\theta_0 \,^\circ F$ is the air temperature and k is a constant.

(ii) Show by integration that the solution of this equation is $\theta = \theta_0 + Ae^{-kt}$, where A is a constant. [5]

The value of θ_0 is 50, and the initial value of θ is 98. The initial rate of temperature loss is 1.5 °F per hour.

- (iii) Find *A*, and show that k = 0.03125. [4]
- (iv) Use this model to calculate how long it will take for the temperature to drop
 - (A) from 98 °F to 89 °F,
 (B) from 98 °F to 80 °F.
 - (*B*) from 98 $^{\circ}$ F to 80 $^{\circ}$ F. [5]
- (v) Comment on the results obtained in parts (i) and (iv). [1]

[Question 8 is printed overleaf.]

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Fig. 8 illustrates a hot air balloon on its side. The balloon is modelled by the volume of revolution 8 about the x-axis of the curve with parametric equations

 $x = 2 + 2\sin\theta$, $y = 2\cos\theta + \sin 2\theta$, $(0 \le \theta \le 2\pi)$.

The curve crosses the x-axis at the point A (4, 0). B and C are maximum and minimum points on the curve. Units on the axes are metres.

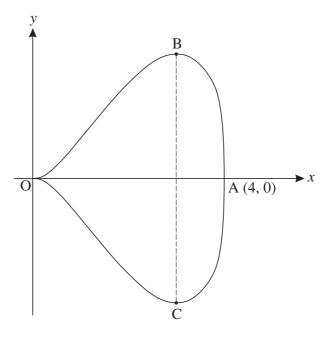


Fig. 8

(i) Find
$$\frac{dy}{dx}$$
 in terms of θ . [4]

(ii) Verify that
$$\frac{dy}{dx} = 0$$
 when $\theta = \frac{1}{6}\pi$, and find the exact coordinates of B

Hence find the maximum width BC of the balloon.

- (iii) (A) Show that $y = x \cos \theta$.
 - (B) Find $\sin \theta$ in terms of x and show that $\cos^2 \theta = x \frac{1}{4}x^2$.
 - (C) Hence show that the cartesian equation of the curve is $y^2 = x^3 \frac{1}{4}x^4$. [7]
- (iv) Find the volume of the balloon.



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[5]

[3]