Section B (36 marks)

7 Scientists can estimate the time elapsed since an animal died by measuring its body temperature.

(i) Assuming the temperature goes down at a constant rate of 1.5 degrees Fahrenheit per hour, estimate how long it will take for the temperature to drop

(*A*) from 98 °F to 89 °F,

(*B*) from 98 °F to 80 °F. **[2]**

In practice, rate of temperature loss is not likely to be constant. A better model is provided by Newton's law of cooling, which states that the temperature θ in degrees Fahrenheit t hours after death is given by the differential equation

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = -k(\theta - \theta_0),$$

where θ_0 °F is the air temperature and k is a constant.

(ii) Show by integration that the solution of this equation is $\theta = \theta_0 + A\mathrm{e}^{-kt}$, where A is a constant. **[5]**

The value of θ_0 is 50, and the initial value of θ is 98. The initial rate of temperature loss is 1.5 °F per hour.

(iii) Find A, and show that $k = 0.031\,25$. **[4]**

(iv) Use this model to calculate how long it will take for the temperature to drop

(*A*) from 98 °F to 89 °F,

(*B*) from 98 °F to 80 °F. **[5]**

(v) Comment on the results obtained in parts **(i)** and **(iv)**. **[1]**

[Question 8 is printed overleaf.]

8 Fig. 8 illustrates a hot air balloon on its side. The balloon is modelled by the volume of revolution about the x-axis of the curve with parametric equations

$$x = 2 + 2\sin\theta, \quad y = 2\cos\theta + \sin 2\theta, \quad (0 \leqslant \theta \leqslant 2\pi).$$

The curve crosses the x-axis at the point A (4, 0). B and C are maximum and minimum points on the curve. Units on the axes are metres.

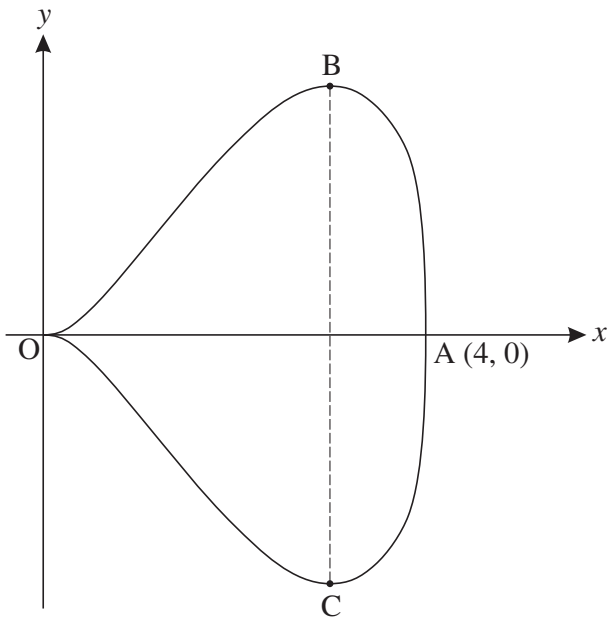

Fig. 8

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of θ. **[4]**

(ii) Verify that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$ when $\theta = \frac{1}{6}\pi$, and find the exact coordinates of B.

Hence find the maximum width BC of the balloon. **[5]**

(iii) (*A*) Show that $y = x\cos\theta$.

(*B*) Find $\sin\theta$ in terms of x and show that $\cos^2\theta = x - \frac{1}{4}x^2$.

(*C*) Hence show that the cartesian equation of the curve is $y^2 = x^3 - \frac{1}{4}x^4$. **[7]**

(iv) Find the volume of the balloon. **[3]**