Section B (36 marks)

7 A glass ornament OABCDEFG is a truncated pyramid on a rectangular base (see Fig. 7). All dimensions are in centimetres.





- (i) Write down the vectors \overrightarrow{CD} and \overrightarrow{CB} . [2]
- (ii) Find the length of the edge CD. [2]
- (iii) Show that the vector $4\mathbf{i} + \mathbf{k}$ is perpendicular to the vectors \overrightarrow{CD} and \overrightarrow{CB} . Hence find the cartesian equation of the plane BCDE. [5]
- (iv) Write down vector equations for the lines OG and AF.

Show that they meet at the point P with coordinates (5, 10, 40). [5]

You may assume that the lines CD and BE also meet at the point P.

The volume of a pyramid is $\frac{1}{3}$ × area of base × height.

(v) Find the volumes of the pyramids POABC and PDEFG.

Hence find the volume of the ornament.

[4]

PMT

8 A curve has equation

$$x^2 + 4y^2 = k^2,$$

where k is a positive constant.

(i) Verify that

$$x = k \cos \theta, \qquad y = \frac{1}{2}k \sin \theta,$$

are parametric equations for the curve.

[3]

- (ii) Hence or otherwise show that $\frac{dy}{dx} = -\frac{x}{4y}$. [3]
- (iii) Fig. 8 illustrates the curve for a particular value of k. Write down this value of k. [1]



Fig. 8

(iv) Copy Fig. 8 and on the same axes sketch the curves for k = 1, k = 3 and k = 4. [3]

On a map, the curves represent the contours of a mountain. A stream flows down the mountain. Its path on the map is always at right angles to the contour it is crossing.

 (\mathbf{v}) Explain why the path of the stream is modelled by the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4y}{x}.$$
[2]

(vi) Solve this differential equation.

Given that the path of the stream passes through the point (2, 1), show that its equation is $y = \frac{x^2}{16}$. [6]

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (OCR) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

OCR is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.