Section B (36 marks)

7 Fig. 7 shows the curve with parametric equations

$$x = \cos\theta,\ y = \sin\theta - \tfrac{1}{8}\sin 2\theta,\ 0 \leqslant \theta < 2\pi.$$

The curve crosses the x-axis at points $A(1, 0)$ and $B(-1, 0)$, and the positive y-axis at C. D is the maximum point of the curve, and E is the minimum point.

The solid of revolution formed when this curve is rotated through 360° about the x-axis is used to model the shape of an egg.

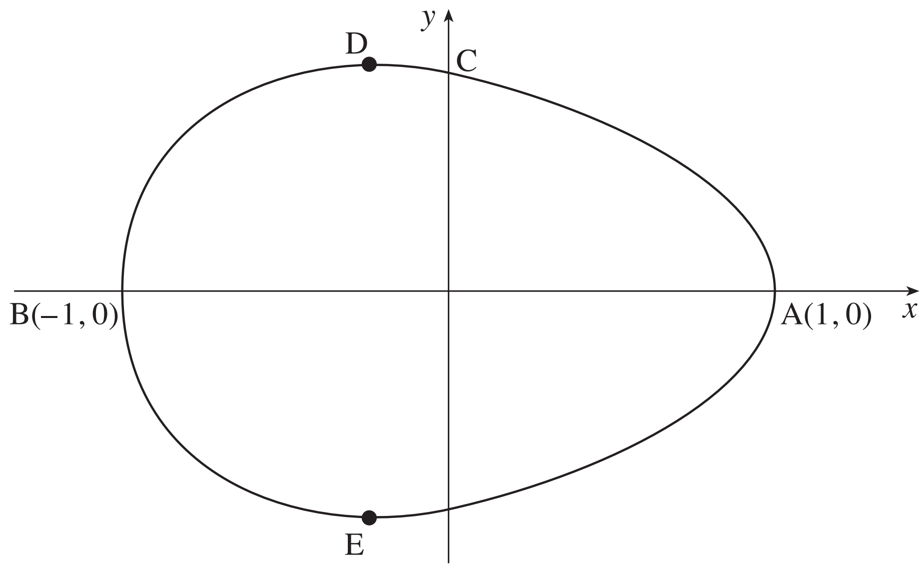

Fig. 7

(i) Show that, at the point A, $\theta = 0$. Write down the value of θ at the point B, and find the coordinates of C. [4]

(ii) Find $\dfrac{dy}{dx}$ in terms of θ.

Hence show that, at the point D,

$$2\cos^2\theta - 4\cos\theta - 1 = 0.$$ [5]

(iii) Solve this equation, and hence find the y-coordinate of D, giving your answer correct to 2 decimal places. [5]

The cartesian equation of the curve (for $0 \leqslant \theta \leqslant \pi$) is

$$y = \tfrac{1}{4}(4 - x)\sqrt{1 - x^2}.$$

(iv) Show that the volume of the solid of revolution of this curve about the x-axis is given by

$$\tfrac{1}{16}\pi \int_{-1}^{1} \left(16 - 8x - 15x^2 + 8x^3 - x^4\right) dx.$$

Evaluate this integral. [6]

8 A pipeline is to be drilled under a river (see Fig. 8). With respect to axes Oxyz, with the x-axis pointing East, the y-axis North and the z-axis vertical, the pipeline is to consist of a straight section AB from the point A$(0, -40, 0)$ to the point B$(40, 0, -20)$ directly under the river, and another straight section BC. All lengths are in metres.

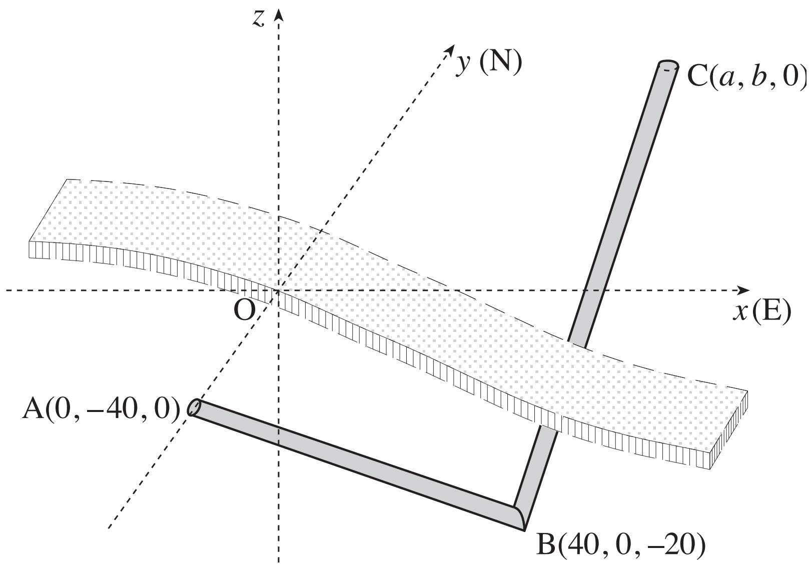

Fig. 8

(i) Calculate the distance AB. [2]

The section BC is to be drilled in the direction of the vector $3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$.

(ii) Find the angle ABC between the sections AB and BC. [4]

The section BC reaches ground level at the point C$(a, b, 0)$.

(iii) Write down a vector equation of the line BC. Hence find a and b. [5]

(iv) Show that the vector $6\mathbf{i} - 5\mathbf{j} + 2\mathbf{k}$ is perpendicular to the plane ABC. Hence find the cartesian equation of this plane. [5]