Paper 1 and Paper 2: Core Pure Mathematics

To support the co-teaching of this qualification with the AS Mathematics qualification, common content has been highlighted in bold.

Topic	What students need to learn:		
	Content		Guidance
2 **Complex numbers**	2.1	**Solve any quadratic equation with real coefficients.** **Solve cubic or quartic equations with real coefficients.**	**Given sufficient information to deduce at least one root for cubics or at least one complex root or quadratic factor for quartics, for example:** **(i)** $\mathrm{f}(z) = 2z^3 - 5z^2 + 7z + 10$ **Given that** $2z - 3$ **is a factor of** $\mathrm{f}(z)$**, use algebra to solve** $\mathrm{f}(z) = 0$ **completely.** **(ii)** $\mathrm{g}(x) = x^4 - x^3 + 6x^2 + 14x - 20$ **Given** $\mathrm{g}(1) = 0$ **and** $\mathrm{g}(-2) = 0$**, use algebra to solve** $\mathrm{g}(x) = 0$ **completely.**
	2.2	**Add, subtract, multiply and divide complex numbers in the form** $x + \mathrm{i}y$ **with** x **and** y **real.** **Understand and use the terms 'real part' and 'imaginary part'.**	**Students should know the meaning of the terms, 'modulus' and 'argument'.**

Topic	What students need to learn:		
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2 **Complex numbers** *continued*	2.3	**Understand and use the complex conjugate.** **Know that non-real roots of polynomial equations with real coefficients occur in conjugate pairs.**	**Knowledge that if z_1 is a root of $\mathrm{f}(z) = 0$ then $z_1{}^*$ is also a root.**
	2.4	**Use and interpret Argand diagrams.**	**Students should be able to represent the sum or difference of two complex numbers on an Argand diagram.**
	2.5	**Convert between the Cartesian form and the modulus-argument form of a complex number.**	Knowledge of radians is assumed.
	2.6	**Multiply and divide complex numbers in modulus argument form.**	**Knowledge of the results** $\lvert z_1 z_2 \rvert = \lvert z_1 \rvert \lvert z_2 \rvert$, $\left\lvert \dfrac{z_1}{z_2} \right\rvert = \dfrac{\lvert z_1 \rvert}{\lvert z_2 \rvert}$ $\arg(z_1 z_2) = \arg z_1 + \arg z_2$ $\arg\left(\dfrac{z_1}{z_2}\right) = \arg z_1 - \arg z_2$ Knowledge of radians and compound angle formulae is assumed.
	2.7	**Construct and interpret simple loci in the argand diagram such as $\lvert z - a \rvert > r$ and $\arg(z - a) = \theta$.**	**To include loci such as $\lvert z - a \rvert = b$, $\lvert z - a \rvert = \lvert z - b \rvert$, $\arg(z - a) = \beta$, and regions such as $\lvert z - a \rvert \le \lvert z - b \rvert$, $\lvert z - a \rvert \le b$, $\alpha < \arg(z - a) < \beta$** Knowledge of radians is assumed.

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2 **Complex numbers** *continued*	2.8	Understand de Moivre's theorem and use it to find multiple angle formulae and sums of series.	To include using the results, $z + \frac{1}{z} = 2\cos\theta$ and $z - \frac{1}{z} = 2\mathrm{i}\sin\theta$ to find $\cos p\theta$, $\sin q\theta$ and $\tan r\theta$ in terms of powers of $\sin\theta$, $\cos\theta$ and $\tan\theta$ and powers of $\sin\theta$, $\cos\theta$ and $\tan\theta$ in terms of multiple angles. For sums of series, students should be able to show that, for example, $1 + z + z^2 + \ldots + z^{n-1} = 1 + \mathrm{i}\cot\left(\frac{\pi}{2n}\right)$ where $z = \cos\left(\frac{\pi}{n}\right) + \mathrm{i}\sin\left(\frac{\pi}{n}\right)$ and n is a positive integer.
	2.9	Know and use the definition $\mathrm{e}^{\mathrm{i}\theta} = \cos\theta + \mathrm{i}\sin\theta$ and the form $z = r\mathrm{e}^{\mathrm{i}\theta}$	Students should be familiar with $\cos\theta = \frac{1}{2}(\mathrm{e}^{\mathrm{i}\theta} + \mathrm{e}^{-\mathrm{i}\theta})$ and $\sin\theta = \frac{1}{2\mathrm{i}}(\mathrm{e}^{\mathrm{i}\theta} - \mathrm{e}^{-\mathrm{i}\theta})$
	2.10	Find the *n* distinct nth roots of $r\mathrm{e}^{\mathrm{i}\theta}$ for $r \neq 0$ and know that they form the vertices of a regular n-gon in the Argand diagram.	
	2.11	Use complex roots of unity to solve geometric problems.	