Leave blank

6. The weights of bags of popcorn are normally distributed with mean of 200 g and 60% of all bags weighing between 190 g and 210 g.

$X \sim N(200, \sigma^2)$

(a) Write down the median weight of the bags of popcorn. **(1)**

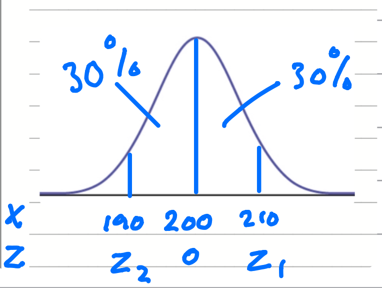
(b) Find the standard deviation of the weights of the bags of popcorn. **(5)**

A shopkeeper finds that customers will complain if their bag of popcorn weighs less than 180 g.

(c) Find the probability that a customer will complain. **(3)**

a) Normal symmetrical so median same as mean = 200g

b)



$z_1 = \Phi^{-1}(0.8) = 0.8416$

$z_2 = \Phi^{-1}(0.2) = -0.8416$

$$Z = \frac{X-\mu}{\sigma}$$

$\sigma Z = X - \mu$ $\qquad \sigma = \dfrac{X-\mu}{Z}$

$$\sigma = \frac{210-200}{0.8416} = 11.88$$

Could also have used $x = 190$ with $z_2 = -0.8416$

c) $X \sim N(\overset{\mu}{200}, \overset{\sigma^2}{11.88^2})$

$P(X < 180) = 0.0461$

M 2 9 2 8 3 A 0 2 0 2 4

Leave blank

7. A packing plant fills bags with cement. The weight X kg of a bag of cement can be modelled by a normal distribution with mean 50 kg and standard deviation 2 kg.

(a) Find P(X>53). **(3)**

$X \sim N(\overset{\mu}{50}, \overset{\sigma^2}{2^2})$

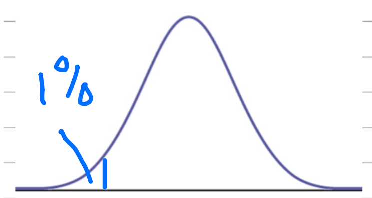
(b) Find the weight that is exceeded by 99% of the bags. **(5)**

Three bags are selected at random.

(c) Find the probability that two weigh more than 53 kg and one weighs less than 53 kg. **(4)**

a) $P(X>53) = 0.0668$

b)



99% exceed 45.35 kg

c) $P(X>53) = 0.0668$ $\quad P(X<53) = 0.9332$

P(2 more and 1 less)

$= 3 \times 0.0668^2 \times 0.9332 = 0.0125$

Multiplied by 3 because 3 ways it could happen MML MLM LMM

M = More L = Less

H32582A02428

Leave blank

6. The random variable X has a normal distribution with mean 30 and standard deviation 5.

(a) Find $P(X < 39)$. **(2)**

$X \sim N(\overset{\mu}{30}, \overset{\sigma^2}{5^2})$

(b) Find the value of d such that $P(X < d) = 0.1151$ **(4)**

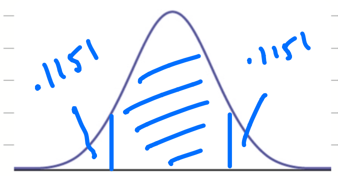
(c) Find the value of e such that $P(X > e) = 0.1151$ **(2)**

(d) Find $P(d < X < e)$. **(2)**

a) $P(X<39) = 0.964$

b) $d = 24$

c) $e = 36$



d) $P(d<X<e) = 1 - 0.1151 - 0.1151$

$= 0.7698$

N32680A02224

Leave blank

8. The lifetimes of bulbs used in a lamp are normally distributed.
A company X sells bulbs with a mean lifetime of 850 hours and a standard deviation of 50 hours.

$X \sim N(\overset{\mu}{850}, \overset{\sigma^2}{50^2})$

(a) Find the probability of a bulb, from company X, having a lifetime of less than 830 hours.
(3)

(b) In a box of 500 bulbs, from company X, find the expected number having a lifetime of less than 830 hours.
(2)

A rival company Y sells bulbs with a mean lifetime of 860 hours and 20% of these bulbs have a lifetime of less than 818 hours.

(c) Find the standard deviation of the lifetimes of bulbs from company Y.
(4)

Both companies sell the bulbs for the same price.

(d) State which company you would recommend. Give reasons for your answer.
(2)

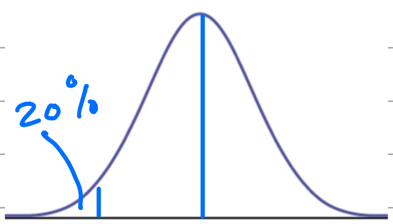
a) $P(X < 830) = 0.3446$

b) $X \sim B(\overset{n}{500}, \overset{p}{0.3446})$

$E(x) = np \quad = 500 \times 0.3446$

$= 172.3$

c)



$X \quad 818 \quad 860$

$z \quad z_1$

$z_1 = \Phi^{-1}(0.2) = -0.8416$

$z = \dfrac{x-\mu}{\sigma} \qquad \sigma = \dfrac{x-\mu}{z}$

$\sigma = \dfrac{818-860}{-0.8416} = 49.90$

d) Recommend Company Y. Their bulbs have a greater mean life and standard deviations are almost the same.

H 3 4 2 7 9 A 0 2 2 2 4

Leave blank

7. The heights of a population of women are normally distributed with mean μ cm and standard deviation σ cm. It is known that 30% of the women are taller than 172 cm and 5% are shorter than 154 cm.

(a) Sketch a diagram to show the distribution of heights represented by this information. **(3)**

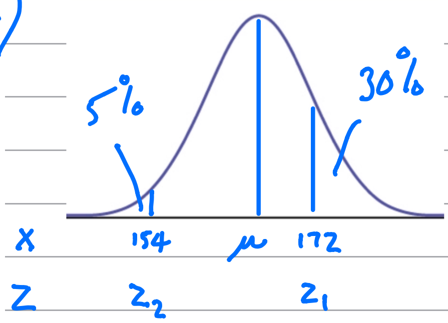
(b) Show that $\mu = 154 + 1.6449\sigma$. **(3)**

(c) Obtain a second equation and hence find the value of μ and the value of σ. **(4)**

A woman is chosen at random from the population.

(d) Find the probability that she is taller than 160 cm. **(3)**

a)



b) $z_1 = \Phi^{-1}(0.7) = 0.5244$

$z_2 = \Phi^{-1}(0.05) = -1.6449$

$Z = \dfrac{X - \mu}{\sigma}$ $\quad \sigma Z = X - \mu$

$\mu = X - \sigma Z$

$\mu = 154 + 1.6449\sigma$ ①

c) Also $\mu = 172 - 0.5244\sigma$ ②

① − ② $\quad 0 = -18 + 2.1693\sigma \quad \Rightarrow \sigma = 8.298$

$\mu = 172 - 0.5244 \times 8.298 = 167.6$

$\mu = 167.6 \quad \sigma = 8.298$

d) $X \sim N(167.6, 8.298^2)$ $\quad P(X > 160)$

$= 0.820$

N 3 5 7 1 1 A 0 2 0 2 4

Leave blank

7. The distances travelled to work, D km, by the employees at a large company are normally distributed with $D \sim \mathrm{N}(30, 8^2)$.

(a) Find the probability that a randomly selected employee has a journey to work of more than 20 km.

(3)

(b) Find the upper quartile, Q_3, of D.

(3)

(c) Write down the lower quartile, Q_1, of D.

(1)

An outlier is defined as any value of D such that $D < h$ or $D > k$ where

$$h = Q_1 - 1.5 \times (Q_3 - Q_1) \quad \text{and} \quad k = Q_3 + 1.5 \times (Q_3 - Q_1)$$

(d) Find the value of h and the value of k.

(2)

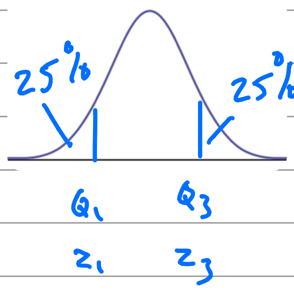
An employee is selected at random.

(e) Find the probability that the distance travelled to work by this employee is an outlier.

(3)

a) $D \sim N(\overset{\mu}{30}, \overset{\sigma^2}{8^2})$ $\quad P(D > 20) = 0.8944$

b)



$z_3 = \Phi^{-1}(0.75) = 0.6749$

$z = \dfrac{x - \mu}{\sigma}$ $\quad x = \sigma z + \mu$

$Q_3 = 8 \times 0.6749 + 30$

$\underline{Q_3 = 35.40}$

c) $\underline{Q_1 = 30 - (35.40 - 30) = 24.60}$

d) $Q_3 - Q_1 = 10.80$

$h = 24.60 - 1.5 \times 10.80 = 8.4$

$k = 35.40 + 1.5 \times 10.80 = 51.6$

H 3 5 3 9 5 A 0 2 4 2 8

Leave blank

Question 7 continued

$D \sim N(30, 8^2)$

e) $P(D > 51.6) = 3.467 \times 10^{-3}$

$P(D < 8.4) = 3.467 \times 10^{-3}$

$P(\text{outlier}) = 3.467 \times 10^{-3} + 3.467 \times 10^{-3}$

$= 6.934 \times 10^{-3}$

$= 0.007$ to 1 s.f.

Q7

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

END

H35395A02728