

Section B

7(i) $\vec{AB} = \begin{pmatrix} -1 \\ -2 \\ 0 \end{pmatrix}$ $\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$	B1 B1 [2]	or equivalent alternative
(ii) $\mathbf{n} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ $\cos \theta = \frac{\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}}{\sqrt{2}\sqrt{5}} = \frac{1}{\sqrt{10}}$ $\Rightarrow \theta = 71.57^\circ$	B1 B1 M1 M1 A1 [5]	correct vectors (any multiples) scalar product used finding invcos of scalar product divided by two modulae 72° or better
(iii) $\cos \phi = \frac{\begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ -2 \\ -1 \end{pmatrix}}{\sqrt{2}\sqrt{9}} = \frac{2+1}{3\sqrt{2}} = \frac{1}{\sqrt{2}}$ $\Rightarrow \phi = 45^\circ *$	M1 A1 E1 [3]	ft their \mathbf{n} for method $\pm 1/\sqrt{2}$ oe exact
(iv) $\sin 71.57^\circ = k \sin 45^\circ$ $\Rightarrow k = \sin 71.57^\circ / \sin 45^\circ = 1.34$	M1 A1 [2]	ft on their 71.57° oe
(v) $\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ -2 \\ -1 \end{pmatrix}$ $x = -2\mu, z = 2 - \mu$ $x + z = -1$ $\Rightarrow -2\mu + 2 - \mu = -1$ $\Rightarrow 3\mu = 3, \mu = 1$ \Rightarrow point of intersection is $(-2, -2, 1)$ distance travelled through glass = distance between $(0, 0, 2)$ and $(-2, -2, 1)$ $= \sqrt{(2^2 + 2^2 + 1^2)} = 3$ cm	M1 M1 A1 A1 B1 [5]	soi subst in $x+z = -1$ www dep on $\mu=1$

<p>8(i) (A) $360^\circ \div 24 = 15^\circ$ $\text{CB}/\text{OB} = \sin 15^\circ$ $\Rightarrow \text{CB} = 1 \sin 15^\circ$ $\Rightarrow \text{AB} = 2\text{CB} = 2 \sin 15^\circ *$</p>	M1 E1 [2]	$\text{AB}=2\text{AC}$ or 2CB $\angle \text{AOC} = 15^\circ$ oe
<p>(B) $\cos 30^\circ = 1 - 2 \sin^2 15^\circ$ $\cos 30^\circ = \sqrt{3}/2$ $\Rightarrow \sqrt{3}/2 = 1 - 2 \sin^2 15^\circ$ $\Rightarrow 2 \sin^2 15^\circ = 1 - \sqrt{3}/2 = (2 - \sqrt{3})/2$ $\Rightarrow \sin^2 15^\circ = (2 - \sqrt{3})/4$ $\Rightarrow \sin 15^\circ = \sqrt{\frac{2 - \sqrt{3}}{4}} = \frac{1}{2}\sqrt{2 - \sqrt{3}} *$</p>	B1 B1 M1 E1 [4]	simplifying
<p>(C) Perimeter = $12 \times \text{AB} = 24 \times \frac{1}{2}\sqrt{(2 - \sqrt{3})}$ $= 12\sqrt{(2 - \sqrt{3})}$ circumference of circle > perimeter of polygon $\Rightarrow 2\pi > 12\sqrt{(2 - \sqrt{3})}$ $\Rightarrow \pi > 6\sqrt{(2 - \sqrt{3})}$</p>	M1 E1 [2]	
<p>(ii) (A) $\tan 15^\circ = \text{FE}/\text{OF}$ $\Rightarrow \text{FE} = \tan 15^\circ$ $\Rightarrow \text{DE} = 2\text{FE} = 2\tan 15^\circ$</p>	M1 E1 [2]	
<p>(B) $\tan 30 = \frac{2 \tan 15}{1 - \tan^2 15} = \frac{2t}{1 - t^2}$ $\tan 30 = 1/\sqrt{3}$ $\Rightarrow \frac{2t}{1 - t^2} = \frac{1}{\sqrt{3}} \Rightarrow 2\sqrt{3}t = 1 - t^2$ $\Rightarrow t^2 + 2\sqrt{3}t - 1 = 0 *$</p>	B1 M1 E1 [3]	
<p>(C) $t = \frac{-2\sqrt{3} \pm \sqrt{12 + 4}}{2} = 2 - \sqrt{3}$ circumference < perimeter $\Rightarrow 2\pi < 24(2 - \sqrt{3})$ $\Rightarrow \pi < 12(2 - \sqrt{3}) *$</p>	M1 A1 M1 E1 [4]	using positive root from exact working
<p>(iii) $6\sqrt{(2 - \sqrt{3})} < \pi < 12(2 - \sqrt{3})$ $\Rightarrow 3.106 < \pi < 3.215$</p>	B1 B1 [2]	3.106, 3.215