

$$7) i) A(1, 2, 2) \\ B(0, 0, 2)$$

$$\vec{AB} = \begin{pmatrix} -1 \\ -2 \\ 0 \end{pmatrix}$$

line AB given by

$$\underline{r} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -2 \\ 0 \end{pmatrix}$$

7ii)

normal is  $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

Find angle between  $\vec{AB}$  and normal

$$\cos\theta = \frac{\begin{pmatrix} -1 \\ -2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}}{\left| \begin{pmatrix} -1 \\ -2 \\ 0 \end{pmatrix} \right| \left| \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right|}$$

$$\cos\theta = \frac{-1 + 0 + 0}{\sqrt{5} \sqrt{2}} = \frac{-1}{\sqrt{10}}$$

$$\theta = 108.4^\circ$$

Acute angle is therefore  $71.6^\circ$

7(iii) Line BC  $\underline{r} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ -2 \\ -1 \end{pmatrix}$  Point of intersection  $(-2, -2, 1)$

$$\cos \phi = \frac{\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ -2 \\ -1 \end{pmatrix}}{\left| \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right| \left| \begin{pmatrix} -2 \\ -2 \\ -1 \end{pmatrix} \right|}$$

$$\cos \phi = \frac{-2 + 0 - 1}{\sqrt{2} \sqrt{9}} = \frac{-3}{3\sqrt{2}}$$

$$\phi = 135^\circ$$

$$\text{Acute angle } \therefore 180 - 135 = 45^\circ$$

7(iv)

$$\sin \theta = k \sin \phi$$

$$\sin 71.6 = k \sin 45$$

$$k = \frac{\sin 71.6^\circ}{\sin 45^\circ} = 1.34$$

7(v)  $\underline{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 - 2\mu \\ 0 - 2\mu \\ 2 - \mu \end{pmatrix}$

$$\text{Subst in plane } x + z = -1$$

$$-2\mu + 2 - \mu = -1$$

$$-3\mu = -3$$

$$\mu = 1$$

Subst back in line

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \\ 1 \end{pmatrix}$$

Distance through glass

$$= \sqrt{(-2-0)^2 + (-2-0)^2 + (1-2)^2} \\ = \sqrt{4+4+1} = 3 \text{ cm}$$

8(i)

$$\text{A}) \angle AOB = \frac{360}{12} = 30^\circ \\ \Rightarrow \angle AOC = 15^\circ$$

$$AC = AO \sin 15^\circ = 1 \sin 15^\circ \\ AB = 2AC = 2 \sin 15^\circ$$

8)

$$\cos 2\theta = 1 - 2 \sin^2 \theta \\ \cos 3\theta = 1 - 2 \sin^2 15^\circ$$

$$\frac{\sqrt{3}}{2} = 1 - 2 \sin^2 15^\circ$$

$$\Rightarrow 2 \sin^2 15^\circ = 1 - \frac{\sqrt{3}}{2} = \frac{2-\sqrt{3}}{2}$$

$$\Rightarrow \sin^2 15^\circ = \frac{2-\sqrt{3}}{4}$$

$$\Rightarrow \sin 15^\circ = \frac{\sqrt{2-\sqrt{3}}}{2}$$

c)

$$\text{Perimeter} = 12AB \\ = 24 \sin 15^\circ$$

$$= 12\sqrt{2-\sqrt{3}}$$

Circumference of circle  
 > perimeter of polygon

$$\therefore 2\pi \times 1 > 12\sqrt{2-\sqrt{3}} \\ \Rightarrow \pi > 6\sqrt{2-\sqrt{3}}$$

8iii)

$$\angle DOE = 30^\circ$$

$$\angle DOF = 15^\circ$$

$$\tan \angle DOF = \tan 15^\circ = \frac{DF}{1}$$

$$\Rightarrow DF = \tan 15^\circ$$

$$DE = 2DF = 2\tan 15^\circ$$


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B)

$$\tan 2\alpha = \frac{2\tan \alpha}{1 - \tan^2 \alpha}$$

$$\tan 30^\circ = \frac{2\tan 15^\circ}{1 - \tan^2 15^\circ} = \frac{2t}{1-t^2}$$

Since  $\tan 30^\circ = \frac{1}{\sqrt{3}}$  we have

$$\frac{1}{\sqrt{3}} = \frac{2t}{1-t^2}$$

$$\Rightarrow 1-t^2 = 2\sqrt{3}t$$

$$t^2 + 2\sqrt{3}t - 1 = 0$$


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c)

$$t = \frac{-2\sqrt{3} \pm \sqrt{12+4}}{2}$$

$$t = \frac{-2\sqrt{3} \pm 4}{2}$$

$$t = -\sqrt{3} + 2 \quad \text{since } t > 0$$

Perimeter of polygon

$$= 12DE = 24\tan 15^\circ$$

$$= 48 - 24\sqrt{3}$$

Circ of circle < perimeter of polygon

$$2\pi \times 1 < 48 - 24\sqrt{3}$$

$$\Rightarrow \pi < 24 - 12\sqrt{3}$$

$$\Rightarrow \pi < 12(2 - \sqrt{3})$$


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8iii)

$$6\sqrt{2-\sqrt{3}} < \pi < 12(2 - \sqrt{3})$$

$$3.1058 < \pi < 3.2154$$


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