

$$C(0, 100, -25)$$

$$0 + 2(100) + 20(-25) + 300 = 0$$

$$200 - 500 + 300 = 0 \quad \checkmark$$

\therefore Plane is the plane ABC

7)

$$\begin{array}{l} \sqrt{3} \sin x - \cos x \\ 2 \left[\frac{\sqrt{3}}{2} \sin x - \frac{1}{2} \cos x \right] \\ = 2 \sin(x - \alpha) \end{array}$$

$$\text{where } \alpha = \tan^{-1} \frac{1}{\sqrt{3}} = \frac{\pi}{6}$$

$$\text{Answer} = 2 \sin\left(x - \frac{\pi}{6}\right)$$

$$\text{Max point when } x - \frac{\pi}{6} = \frac{\pi}{2}$$

$$\Rightarrow x = \frac{\pi}{2} + \frac{\pi}{6} = \frac{2\pi}{3}$$

$$\text{Max at } \left(\frac{2\pi}{3}, 2\right)$$

8)

Subst A, B, C in eqn of plane
i) to verify it is plane ABC

$$3x + 2y + 20z + 300 = 0$$

$$A(0, 0, -15)$$

$$0 + 0 + 20(-15) + 300 = 0 \quad \checkmark$$

$$B(100, 0, -30)$$

$$300 + 0 + 20(-30) + 300 = 0$$

$$300 - 600 + 300 = 0 \quad \checkmark$$

ii)

$$D(0, 0, -40)$$

$$E(100, 0, -50)$$

$$F(0, 100, -35)$$

$$\vec{DE} = \begin{pmatrix} 100 \\ 0 \\ -10 \end{pmatrix} \quad \vec{DF} = \begin{pmatrix} 0 \\ 100 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} 100 \\ 0 \\ -10 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 20 \end{pmatrix} = 200 - 200 = 0$$

$$\begin{pmatrix} 0 \\ 100 \\ 5 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 20 \end{pmatrix} = -100 + 100 = 0$$

$\therefore \begin{pmatrix} 2 \\ -1 \\ 20 \end{pmatrix}$ is \perp to both \vec{DE} and \vec{DF}

It is \therefore normal to plane DEF

Plane is of form

$$2x - y + 20z = d$$

D(0, 0, -40) on plane so

$$0 - 0 + 20(-40) = d$$

$$-800 = d$$

Plane DEF is

$$2x - y + 20z = -800$$

8iii) Angle between planes
= angle between their normals

$$\cos \theta = \frac{\left(\begin{array}{c} 3 \\ 2 \\ 20 \end{array} \right) \cdot \left(\begin{array}{c} 2 \\ -1 \\ 20 \end{array} \right)}{\left| \left(\begin{array}{c} 3 \\ 2 \\ 20 \end{array} \right) \right| \left| \left(\begin{array}{c} 2 \\ -1 \\ 20 \end{array} \right) \right|}$$

$$\cos \theta = \frac{6 - 2 + 400}{\sqrt{9+4+400} \sqrt{4+1+400}}$$

$$\cos \theta = \frac{404}{\sqrt{413} \sqrt{405}}$$

$$\theta = 8.95^\circ$$

8iv)

$$\underline{r} = \left(\begin{array}{c} 15 \\ 34 \\ 0 \end{array} \right) + \lambda \left(\begin{array}{c} 3 \\ 2 \\ 20 \end{array} \right)$$

is eqn of line RS

At S

$$\left(\begin{array}{c} x \\ y \\ z \end{array} \right) = \left(\begin{array}{c} 15 + 3\lambda \\ 34 + 2\lambda \\ 0 + 20\lambda \end{array} \right)$$

S on plane ABC so

$$3(15 + 3\lambda) + 2(34 + 2\lambda)$$

$$+ 20(20\lambda) + 300 = 0$$

$$45 + 9\lambda + 68 + 4\lambda + 400\lambda + 300 = 0$$

$$413\lambda = -413$$

$$\lambda = -1$$

$$\underline{r}_S = \left(\begin{array}{c} 15 - 3 \\ 34 - 2 \\ -20 \end{array} \right) = \left(\begin{array}{c} 12 \\ 32 \\ -20 \end{array} \right)$$

S is point (12, 32, -20)

9)

$$\text{i) } \frac{dv}{dt} = 10e^{-\frac{1}{2}t}$$

$$v = \int 10e^{-\frac{1}{2}t} dt$$

$$v = -20e^{-\frac{1}{2}t} + C$$

$$t=0, v=0$$

$$0 = -20 + C$$

$$20 = C$$

$$\text{ii) } \therefore v = -20e^{-\frac{1}{2}t} + 20 \text{ ms}^{-1}$$

$$v \rightarrow 20 \text{ ms}^{-1} \text{ as } t \rightarrow \infty$$

iii)

$$\text{Let } \frac{1}{(w-4)(w+5)} = \frac{A}{w-4} + \frac{B}{w+5}$$

$$\Rightarrow 1 \equiv A(w+5) + B(w-4)$$

$$\text{when } w=-5 \quad 1 = B(-5-4)$$

$$\Rightarrow B = -\frac{1}{9}$$

$$\text{when } w=4 \quad 1 = A(4+5)$$

$$\Rightarrow A = \frac{1}{9}$$

$$\frac{1}{(w-4)(w+5)} = \frac{1}{9(w-4)} - \frac{1}{9(w+5)}$$

$$\text{iv) } \frac{dw}{dt} = -\frac{1}{2}(w-4)(w+5)$$

$$\Rightarrow w-4 \rightarrow 0$$

$$\Rightarrow w \rightarrow 4 \text{ ms}^{-1}$$

$$\Rightarrow \int \frac{1}{(w-4)(w+5)} dw = \int -\frac{1}{2} dt$$

$$\Rightarrow \int \left(\frac{1}{9(w-4)} - \frac{1}{9(w+5)} \right) dw = \int -\frac{1}{2} dt$$

$$\Rightarrow \frac{1}{9} \ln(w-4) - \frac{1}{9} \ln(w+5) = -\frac{1}{2} t + c$$

$$\Rightarrow \frac{1}{9} \ln\left(\frac{w-4}{w+5}\right) = -\frac{1}{2} t + c$$

$$\Rightarrow \ln\left(\frac{w-4}{w+5}\right) = -\frac{9t}{2} + c$$

$$\Rightarrow \frac{w-4}{w+5} = e^{-\frac{9t}{2} + c}$$

$$\Rightarrow \frac{w-4}{w+5} = Ae^{-\frac{9t}{2}}$$

Given $w = 10$ when $t = 0$

$$\therefore \frac{10-4}{10+5} = Ae^0$$

$$\Rightarrow \frac{6}{15} = A$$

$$\Rightarrow 0.4 = A$$

$$\therefore \frac{w-4}{w+5} = 0.4e^{-4.5t}$$

v) As $t \rightarrow \infty$

$$\frac{w-4}{w+5} \rightarrow 0$$