

Section B

<p>8(i) At A: $3 \times 0 + 2 \times 0 + 20 \times (-15) + 300 = 0$ At B: $3 \times 100 + 2 \times 0 + 20 \times (-30) + 300 = 0$ At C: $3 \times 0 + 2 \times 100 + 20 \times (-25) + 300 = 0$ So ABC has equation $3x + 2y + 20z + 300 = 0$</p>	M1 A2,1,0 [3]	substituting co-ords into equation of plane... for ABC OR using two vectors in the plane form vector product M1A1 then $3x + 2y + 20z = c = -300$ A1 OR using vector equation of plane M1, elim both parameters M1, A1
<p>(ii) $\overrightarrow{DE} = \begin{pmatrix} 100 \\ 0 \\ -10 \end{pmatrix}$ $\overrightarrow{DF} = \begin{pmatrix} 0 \\ 100 \\ 5 \end{pmatrix}$</p> $\begin{pmatrix} 100 \\ 0 \\ -10 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 20 \end{pmatrix} = 100 \times 2 + 0 \times -1 + -10 \times 20 = 200 - 200 = 0$ $\begin{pmatrix} 0 \\ 100 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 20 \end{pmatrix} = 0 \times 2 + 100 \times -1 + 5 \times 20 = -100 + 100 = 0$	B1B1 B1 B1	need evaluation need evaluation
Equation of plane is $2x - y + 20z = c$ At D (say) $c = 20 \times -40 = -800$ So equation is $2x - y + 20z + 800 = 0$	M1 A1 [6]	
<p>(iii) Angle is θ, where</p> $\cos \theta = \frac{\begin{pmatrix} 2 \\ -1 \\ 20 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \\ 20 \end{pmatrix}}{\sqrt{2^2 + (-1)^2 + 20^2} \sqrt{3^2 + 2^2 + 20^2}} = \frac{404}{\sqrt{405} \sqrt{413}}$ $\Rightarrow \theta = 8.95^\circ$	M1 A1 A1 A1cao [4]	formula with correct vectors top bottom (or 0.156 radians)
<p>(iv) RS: $\mathbf{r} = \begin{pmatrix} 15 \\ 34 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 2 \\ 20 \end{pmatrix}$</p> $= \begin{pmatrix} 15+3\lambda \\ 34+2\lambda \\ 20\lambda \end{pmatrix}$ $\Rightarrow 3(15+3\lambda) + 2(34+2\lambda) + 20.20\lambda + 300 = 0$ $\Rightarrow 45 + 9\lambda + 68 + 4\lambda + 400\lambda + 300 = 0$ $\Rightarrow 413 + 413\lambda = 0$ $\Rightarrow \lambda = -1$ so S is $(12, 32, -20)$	B1 B1 M1 A1 A1 [5]	$\begin{pmatrix} 15 \\ 34 \\ 0 \end{pmatrix} + \dots$ $\dots + \lambda \begin{pmatrix} 3 \\ 2 \\ 20 \end{pmatrix}$ solving with plane $\lambda = -1$ cao

<p>9(i)</p> $v = \int 10e^{-\frac{1}{2}t} dt$ $= -20e^{-\frac{1}{2}t} + c$ <p>when $t = 0, v = 0$</p> $\Rightarrow 0 = -20 + c$ $\Rightarrow c = 20$ <p>so $v = 20 - 20e^{-\frac{1}{2}t}$</p>	M1 A1 M1 A1 [4]	separate variables and intend to integrate $-20e^{-\frac{1}{2}t}$ finding c cao
<p>(ii) As $t \rightarrow \infty e^{-1/2t} \rightarrow 0$</p> $\Rightarrow v \rightarrow 20$ <p>So long term speed is 20 m s^{-1}</p>	M1 A1 [2]	ft (for their $c > 0$, found)
<p>(iii)</p> $\frac{1}{(w-4)(w+5)} = \frac{A}{w-4} + \frac{B}{w+5}$ $= \frac{A(w+5) + B(w-4)}{(w-4)(w+5)}$ $\Rightarrow 1 \equiv A(w+5) + B(w-4)$ <p>$w = 4: 1 = 9A \Rightarrow A = 1/9$</p> <p>$w = -5: 1 = -9B \Rightarrow B = -1/9$</p> $\Rightarrow \frac{1}{(w-4)(w+5)} = \frac{1/9}{w-4} - \frac{1/9}{w+5}$ $= \frac{1}{9(w-4)} - \frac{1}{9(w+5)}$	M1 M1 A1 A1 [4]	cover up, substitution or equating coeffs 1/9 -1/9
<p>(iv)</p> $\frac{dw}{dt} = -\frac{1}{2}(w-4)(w+5)$ $\Rightarrow \int \frac{dw}{(w-4)(w+5)} = \int -\frac{1}{2} dt$ $\Rightarrow \int \left[\frac{1}{9(w-4)} - \frac{1}{9(w+5)} \right] dw = \int -\frac{1}{2} dt$ $\Rightarrow \frac{1}{9} \ln(w-4) - \frac{1}{9} \ln(w+5) = -\frac{1}{2} t + c$ $\Rightarrow \frac{1}{9} \ln \frac{w-4}{w+5} = -\frac{1}{2} t + c$ <p>When $t = 0, w = 10 \Rightarrow c = \frac{1}{9} \ln \frac{6}{15} = \frac{1}{9} \ln \frac{2}{5}$</p> $\Rightarrow \ln \frac{w-4}{w+5} = -\frac{9}{2} t + \ln \frac{2}{5}$ $\Rightarrow \frac{w-4}{w+5} = e^{\frac{-9}{2}t + \ln \frac{2}{5}} = \frac{2}{5} e^{\frac{-9}{2}t} = 0.4 e^{-4.5t} *$	M1 M1 A1 ft M1 M1 E1 [6]	separating variables substituting their partial fractions integrating correctly (condone absence of c) correctly evaluating c (at any stage) combining lns (at any stage) www
<p>(v) As $t \rightarrow \infty e^{-4.5t} \rightarrow 0$</p> $\Rightarrow w-4 \rightarrow 0$ <p>So long term speed is 4 m s^{-1}.</p>	M1 A1 [2]	