

Section B

7 (a) (i) $P_{\max} = \frac{2}{2-1} = 2$ $P_{\min} = \frac{2}{2+1} = 2/3.$	B1 B1 [2]	
(ii) $P = \frac{2}{2-\sin t} = 2(2-\sin t)^{-1}$ $\Rightarrow \frac{dP}{dt} = -2(2-\sin t)^{-2} \cdot -\cos t$ $= \frac{2\cos t}{(2-\sin t)^2}$ $\frac{1}{2}P^2 \cos t = \frac{1}{2} \frac{4}{(2-\sin t)^2} \cos t$ $= \frac{2\cos t}{(2-\sin t)^2} = \frac{dP}{dt}$	M1 B1 A1 DM1 E1 [5]	chain rule $-1(\dots)^{-2}$ soi (or quotient rule M1,numerator A1,denominator A1) attempt to verify or by integration as in (b)(ii)
(b)(i) $\frac{1}{P(2P-1)} = \frac{A}{P} + \frac{B}{2P-1}$ $= \frac{A(2P-1)+BP}{P(2P-1)}$ $\Rightarrow 1 = A(2P-1) + BP$ $P=0 \Rightarrow 1 = -A \Rightarrow A = -1$ $P = \frac{1}{2} \Rightarrow 1 = -A + \frac{1}{2}B \Rightarrow B = 2$ So $\frac{1}{P(2P-1)} = -\frac{1}{P} + \frac{2}{2P-1}$	M1 M1 A1 A1 [4]	correct partial fractions substituting values, equating coeffs or cover up rule $A = -1$ $B = 2$
(ii) $\frac{dP}{dt} = \frac{1}{2}(2P-P^2)\cos t$ $\Rightarrow \int \frac{1}{2P^2-P} dP = \int \frac{1}{2} \cos t dt$ $\Rightarrow \int \left(\frac{2}{2P-1} - \frac{1}{P} \right) dP = \int \frac{1}{2} \cos t dt$ $\Rightarrow \ln(2P-1) - \ln P = \frac{1}{2} \sin t + c$ When $t=0, P=1$ $\Rightarrow \ln 1 - \ln 1 = \frac{1}{2} \sin 0 + c \Rightarrow c=0$ $\Rightarrow \ln\left(\frac{2P-1}{P}\right) = \frac{1}{2} \sin t *$	M1 A1 A1 B1 E1 [5]	separating variables $\ln(2P-1) - \ln P$ ft their A,B from (i) $\frac{1}{2} \sin t$ finding constant = 0
(iii) $P_{\max} = \frac{1}{2-e^{1/2}} = 2.847$ $P_{\min} = \frac{1}{2-e^{-1/2}} = 0.718$	M1A1 M1A1 [4]	www www

<p>8 (i)</p> $\frac{dy}{dx} = \frac{10\cos\theta + 10\cos 2\theta}{-10\sin\theta - 10\sin 2\theta}$ $= -\frac{\cos\theta + \cos 2\theta}{\sin\theta + \sin 2\theta} *$ <p>When $\theta = \pi/3$, $\frac{dy}{dx} = -\frac{\cos\pi/3 + \cos 2\pi/3}{\sin\pi/3 + \sin 2\pi/3}$ $= 0$ as $\cos\pi/3 = \frac{1}{2}$, $\cos 2\pi/3 = -\frac{1}{2}$</p> <p>At A $x = 10 \cos \pi/3 + 5 \cos 2\pi/3$ $= 2\frac{1}{2}$ $y = 10 \sin \pi/3 + 5 \sin 2\pi/3 = 15\sqrt{3}/2$</p>	M1 E1 B1	$dy/d\theta \div dx/d\theta$ or solving $\cos\theta + \cos 2\theta = 0$ M1 A1 A1 [6]
<p>(ii)</p> $x^2 + y^2 = (10\cos\theta + 5\cos 2\theta)^2 + (10\sin\theta + 5\sin 2\theta)^2$ $= 100\cos^2\theta + 100\cos\theta \cos 2\theta + 25\cos^2 2\theta$ $+ 100\sin^2\theta + 100\sin\theta \sin 2\theta + 25\sin^2 2\theta$ $= 100 + 100\cos(2\theta - \theta) + 25$ $= 125 + 100\cos\theta *$	B1 M1 DM1 E1 [4]	expanding cos 2θ cos θ + sin 2θ sin θ = cos(2θ - θ) or substituting for sin 2θ and cos 2θ B1 B1 [2]
<p>(iii)</p> $\text{Max } \sqrt{125+100} = 15$ $\text{min } \sqrt{125-100} = 5$		
<p>(iv)</p> $2\cos^2\theta + 2\cos\theta - 1 = 0$ $\cos\theta = \frac{-2 \pm \sqrt{12}}{4} = \frac{-2 \pm 2\sqrt{3}}{4}$ <p>At B, $\cos\theta = \frac{-1 + \sqrt{3}}{2}$</p> $\text{OB}^2 = 125 + 50(-1 + \sqrt{3}) = 75 + 50\sqrt{3} = 161.6\dots$ $\Rightarrow \text{OB} = \sqrt{161.6\dots} = 12.7 \text{ (m)}$	M1 A1 M1 A1 [4]	quadratic formula or $\theta = 68.53^\circ$ or 1.20 radians, correct root selected or $\text{OB} = 10\sin\theta + 5\sin 2\theta$ ft their $\theta/\cos\theta$ oe cao