

Section B

<p>6 (i) At E, $\theta = 2\pi$ $\Rightarrow x = a(2\pi - \sin 2\pi) = 2a\pi$ So OE = $2a\pi$. Max height is when $\theta = \pi$ $\Rightarrow y = a(1 - \cos \pi) = 2a$</p>	M1 A1 M1 A1 [4]	$\theta = \pi, 180^\circ, \cos \theta = -1$
<p>(ii) $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ $= \frac{a \sin \theta}{a(1 - \cos \theta)}$ $= \frac{\sin \theta}{(1 - \cos \theta)}$</p>	M1 M1 A1 [3]	$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ for theirs $\frac{d}{d\theta}(\sin \theta) = \cos \theta, \frac{d}{d\theta}(\cos \theta) = -\sin \theta$ both or equivalent www condone uncancelled a
<p>(iii) $\tan 30^\circ = 1/\sqrt{3}$ $\Rightarrow \frac{\sin \theta}{(1 - \cos \theta)} = \frac{1}{\sqrt{3}}$ $\Rightarrow \sin \theta = \frac{1}{\sqrt{3}}(1 - \cos \theta)^*$ When $\theta = 2\pi/3$, $\sin \theta = \sqrt{3}/2$ $(1 - \cos \theta)/\sqrt{3} = (1 + \frac{1}{2})/\sqrt{3} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$ BF = $a(1 + \frac{1}{2}) = 3a/2^*$ OF = $a(2\pi/3 - \sqrt{3}/2)$</p>	M1 E1 M1 E1 E1 B1 [6]	Or gradient = $1/\sqrt{3}$ $\sin \theta = \sqrt{3}/2, \cos \theta = -1/2$ or equiv.
<p>(iv) BC = $2a\pi - 2a(2\pi/3 - \sqrt{3}/2)$ = $a(2\pi/3 + \sqrt{3})$ AF = $\sqrt{3} \times 3a/2 = 3\sqrt{3}a/2$ AD = BC + 2AF = $a(2\pi/3 + \sqrt{3} + 3\sqrt{3})$ = $a(2\pi/3 + 4\sqrt{3})$ = 20 $\Rightarrow a = 2.22$ m</p>	B1ft M1 A1 M1 A1 [5]	their OE - 2 their OF

7 (i) $AE = \sqrt{(15^2 + 20^2 + 0^2)} = 25$	M1 A1 [2]	
(ii) $\overrightarrow{AE} = \begin{pmatrix} 15 \\ -20 \\ 0 \end{pmatrix} = 5 \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix}$ Equation of BD is $\mathbf{r} = \begin{pmatrix} -1 \\ -7 \\ 11 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix}$ $BD = 15 \Rightarrow \lambda = 3$ $\Rightarrow D \text{ is } (8, -19, 11)$	M1 A1 M1 A1cao [4]	Any correct form or $\mathbf{r} = \begin{pmatrix} -1 \\ -7 \\ 11 \end{pmatrix} + \lambda \begin{pmatrix} 15 \\ -20 \\ 0 \end{pmatrix}$ $\lambda = 3$ or $3/5$ as appropriate
(iii) At A: $-3 \times 0 + 4 \times 0 + 5 \times 6 = 30$ At B: $-3 \times (-1) + 4 \times (-7) + 5 \times 11 = 30$ At C: $-3 \times (-8) + 4 \times (-6) + 5 \times 6 = 30$ Normal is $\begin{pmatrix} -3 \\ 4 \\ 5 \end{pmatrix}$	M1 A2,1,0 B1 [4]	One verification (OR B1 Normal, M1 scalar product with 1 vector in the plane, A1 two correct, A1 verification with a point OR M1 vector form of equation of plane eg $\mathbf{r} = 0\mathbf{i} + 0\mathbf{j} + 6\mathbf{k} + \mu(\mathbf{i} + 7\mathbf{j} - 5\mathbf{k}) + \nu(8\mathbf{i} + 6\mathbf{j} + 0\mathbf{k})$ M1 elimination of both parameters A1 equation of plane B1 Normal *)
(iv) $\begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix} \cdot \overrightarrow{AE} = \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 15 \\ -20 \\ 0 \end{pmatrix} = 60 - 60 = 0$ $\begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix} \cdot \overrightarrow{AB} = \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -7 \\ 5 \end{pmatrix} = -4 - 21 + 25 = 0$ $\Rightarrow \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix} \text{ is normal to plane}$ Equation is $4x + 3y + 5z = 30$.	M1 E1 M1 A1 [4]	scalar product with one vector in plane = 0 scalar product with another vector in plane = 0 $4x + 3y + 5z = \dots$ 30 OR as * above OR M1 for subst 1 point in $4x+3y+5z=$, A1 for subst 2 further points =30 A1 correct equation, B1 Normal
(v) Angle between planes is angle between normals $\begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ 4 \\ 5 \end{pmatrix}$ $\cos \theta = \frac{4 \times (-3) + 3 \times 4 + 5 \times 5}{\sqrt{50} \times \sqrt{50}} = \frac{1}{2}$ $\Rightarrow \theta = 60^\circ$	M1 M1 A1 A1 [4]	Correct method for any 2 vectors their normals only (rearranged) or 120° cao