SECTION B

7 (i) $\int \frac{t}{1+t^2}dt = ½ \ln(1 + t^2) + c$ **OR** $\int \frac{t}{1+t^2}dt$ let $u = 1 + t^2$, $du = 2t\,dt$ $= \int \frac{1/2}{u}du$ $= ½ \ln u + c$ $= ½ \ln(1 + t^2) + c$	**M1** **A2** **M1** **A1** **A1** **[3]**	$k \ln(1 + t^2)$ $½ \ln(1 + t^2)$ [$+c$] substituting $u = 1 + t^2$ condone no c
(ii) $\frac{1}{t(1+t^2)} = \frac{A}{t} + \frac{Bt+C}{1+t^2}$ $\Rightarrow$ $1 = A(1+ t^2) + (Bt + C)t$ $t = 0 \Rightarrow 1 = A$ coeff^t of t^2 $\Rightarrow 0 = A + B$ $\Rightarrow B = -1$ coeff^t of t $\Rightarrow 0 = C$ $\Rightarrow$ $\frac{1}{t(1+t^2)} = \frac{1}{t} - \frac{t}{1+t^2}$	 **M1** **M1** **A1** **A1** **A1** **[5]**	 Equating numerators substituting or equating coeff^{t}s dep 1st M1 $A = 1$ $B = -1$ $C = 0$
(iii) $\frac{dM}{dt} = \frac{M}{t(1+t^2)}$ $\Rightarrow$ $\int \frac{1}{M}dM = \int \frac{1}{t(1+t^2)}dt = \int [\frac{1}{t} - \frac{t}{1+t^2}]dt$ $\Rightarrow$ $\ln M = \ln t - ½ \ln(1 + t^2) + c$ $= \ln(\frac{e^c t}{\sqrt{1+t^2}})$ $\Rightarrow$ $M = \frac{Kt}{\sqrt{1+t^2}}$ * where $K = e^c$	 **M1** **B1** **A1ft** **M1** **M1** **E1** **[6]**	 Separating variables and substituting their partial fractions $\ln M = \ldots$ $\ln t - ½ \ln(1 + t^2) + c$ combining $\ln t$ and $½ \ln(1 + t^2)$ $K = e^c$ o.e.
(iv) $t = 1$, $M = 25 \Rightarrow 25 = K/\sqrt{2}$ $\Rightarrow$ $K = 25\sqrt{2} = 35.36$ As $t \to \infty$, $M \to K$ So long term value of M is 35.36 grams	**M1** **A1** **M1** **A1ft** **[4]**	 $25\sqrt{2}$ or 35 or better soi ft their K.
8 (i) P is (0, 10, 30) Q is (0, 20, 15) R is (–15, 20, 30) $\Rightarrow$ $\overrightarrow{PQ} = \begin{pmatrix} 0-0 \\ 20-10 \\ 15-30 \end{pmatrix} = \begin{pmatrix} 0 \\ 10 \\ -15 \end{pmatrix}$ * $\Rightarrow$ $\overrightarrow{PR} = \begin{pmatrix} -15-0 \\ 20-10 \\ 30-30 \end{pmatrix} = \begin{pmatrix} -15 \\ 10 \\ 0 \end{pmatrix}$ *	 **B2,1,0** **E1** **E1** **[4]**	

(ii) $\begin{pmatrix}2\\3\\2\end{pmatrix}.\begin{pmatrix}0\\10\\-15\end{pmatrix}=0+30-30=0$	**M1**	Scalar product with 1 vector in the plane OR vector x product oe
$\begin{pmatrix}2\\3\\2\end{pmatrix}.\begin{pmatrix}-15\\10\\0\end{pmatrix}=-30+30+0=0$	**E1**	
$\Rightarrow$ $\begin{pmatrix}2\\3\\2\end{pmatrix}$ is normal to the plane		
$\Rightarrow$ equation of plane is $2x + 3y + 2z = c$	**M1**	$2x + 3y + 2z = c$ **or** an appropriate vector form
At P (say), $x = 0$, $y = 10$, $z = 30$ $\Rightarrow$ $c = 2\times0 + 3\times10 + 2\times30 = 90$	**M1dep**	substituting to find c **or** completely eliminating parameters
$\Rightarrow$ equation of plane is $2x + 3y + 2z = 90$	**A1 cao** **[5]**	
(iii) S is $(-7\frac{1}{2}, 20, 22\frac{1}{2})$	**B1**	
$\overrightarrow{OT}=\overrightarrow{OP}+\frac{2}{3}\overrightarrow{PS}$	**M1**	**Or** $\frac{1}{3}(\overrightarrow{OP}+\overrightarrow{OR}+\overrightarrow{OQ})$ oe ft their S
$=\begin{pmatrix}0\\10\\30\end{pmatrix}+\frac{2}{3}\begin{pmatrix}-7\frac{1}{2}\\10\\-7\frac{1}{2}\end{pmatrix}=\begin{pmatrix}-5\\16\frac{2}{3}\\25\end{pmatrix}$	**A1ft**	**Or** $\frac{1}{3}\begin{pmatrix}0\\10\\30\end{pmatrix}+\frac{2}{3}\begin{pmatrix}-7\frac{1}{2}\\20\\22\frac{1}{2}\end{pmatrix}$ ft their S
So T is $(-5, 16\frac{2}{3}, 25)$ *	**E1** **[4]**	
(iv) $\mathbf{r}=\begin{pmatrix}-5\\16\frac{2}{3}\\25\end{pmatrix}+\lambda\begin{pmatrix}2\\3\\2\end{pmatrix}$	**B1,B1**	$\begin{pmatrix}-5\\16\frac{2}{3}\\25\end{pmatrix}+\ldots\ldots+\lambda\begin{pmatrix}2\\3\\2\end{pmatrix}$
At C (–30, 0, 0): $-5 + 2\lambda = -30$, $16\frac{2}{3}+3\lambda=0$, $25 + 2\lambda = 0$	**M1** **A1**	Substituting coordinates of C into vector equation At least 2 relevant correct equations for λ
1st and 3rd eqns give $\lambda = -12\frac{1}{2}$, not compatible with 2nd. So line does not pass through C.	**E1** **[5]**	oe www