

6	(i)	$\begin{aligned} BAC &= 120 - 90 - (90 - \theta) \\ &= \theta - 60 \\ \Rightarrow BC &= b \sin(\theta - 60) \\ CD &= AE = a \sin \theta \\ \Rightarrow h &= BC + CD = a \sin \theta + b \sin(\theta - 60^\circ) * \end{aligned}$	B1 M1 E1 [3]	
	(ii)	$\begin{aligned} h &= a \sin \theta + b \sin(\theta - 60^\circ) \\ &= a \sin \theta + b (\sin \theta \cos 60 - \cos \theta \sin 60) \\ &= a \sin \theta + \frac{1}{2} b \sin \theta - \frac{\sqrt{3}}{2} b \cos \theta \\ &= \left(a + \frac{1}{2} b\right) \sin \theta - \frac{\sqrt{3}}{2} b \cos \theta * \end{aligned}$	M1 M1 E1 [3]	corr compound angle formula $\sin 60 = \sqrt{3}/2$, $\cos 60 = 1/2$ used
	(iii)	$\begin{aligned} OB \text{ horizontal when } h &= 0 \\ \Rightarrow \left(a + \frac{1}{2} b\right) \sin \theta - \frac{\sqrt{3}}{2} b \cos \theta &= 0 \\ \Rightarrow \left(a + \frac{1}{2} b\right) \sin \theta &= \frac{\sqrt{3}}{2} b \cos \theta \\ \Rightarrow \frac{\sin \theta}{\cos \theta} &= \frac{\frac{\sqrt{3}}{2} b}{a + \frac{1}{2} b} \\ \Rightarrow \tan \theta &= \frac{\sqrt{3}b}{2a+b} * \end{aligned}$	M1 M1 E1 [3]	$\frac{\sin \theta}{\cos \theta} = \tan \theta$
	(iv)	$\begin{aligned} 2 \sin \theta - \sqrt{3} \cos \theta &= R \sin(\theta - \alpha) \\ &= R(\sin \theta \cos \alpha - \cos \theta \sin \alpha) \\ \Rightarrow R \cos \alpha &= 2, R \sin \alpha = \sqrt{3} \\ \Rightarrow R^2 &= 2^2 + (\sqrt{3})^2 = 7, R = \sqrt{7} = 2.646 \text{ m} \\ \tan \alpha &= \sqrt{3}/2, \alpha = 40.9^\circ \\ \text{So } h &= \sqrt{7} \sin(\theta - 40.9^\circ) \\ \Rightarrow h_{\max} &= \sqrt{7} = 2.646 \text{ m} \\ \text{when } \theta - 40.9^\circ &= 90^\circ \\ \Rightarrow \theta &= 130.9^\circ \end{aligned}$	M1 B1 M1A1 B1ft M1 A1 [7]	

7	<p>(i)</p> $\frac{dx}{dt} = -1(1+e^{-t})^{-2} \cdot -e^{-t}$ $= \frac{e^{-t}}{(1+e^{-t})^2}$ $1-x = 1 - \frac{1}{1+e^{-t}}$ $1-x = \frac{1+e^{-t}-1}{1+e^{-t}} = \frac{e^{-t}}{1+e^{-t}}$ $\Rightarrow x(1-x) = \frac{1}{1+e^{-t}} \cdot \frac{e^{-t}}{1+e^{-t}} = \frac{e^{-t}}{(1+e^{-t})^2}$ $\Rightarrow \frac{dx}{dt} = x(1-x)$ <p>When $t = 0$, $x = \frac{1}{1+e^0} = 0.5$</p>	M1 A1 M1 A1 E1 B1 [6]	chain rule substituting for $x(1-x)$ $1-x = \frac{1+e^{-t}-1}{1+e^{-t}} = \frac{e^{-t}}{1+e^{-t}}$ [OR, M1 A1 for solving differential equation for t , B1 use of initial condition, M1 A1 making x the subject, E1 required form]
	<p>(ii)</p> $\frac{1}{(1+e^{-t})} = \frac{3}{4}$ $\Rightarrow e^{-t} = 1/3$ $\Rightarrow t = -\ln 1/3 = 1.10 \text{ years}$	M1 M1 A1 [3]	correct log rules
	<p>(iii)</p> $\frac{1}{x^2(1-x)} = \frac{A}{x^2} + \frac{B}{x} + \frac{C}{1-x}$ $\Rightarrow 1 = A(1-x) + Bx(1-x) + Cx^2$ $x = 0 \Rightarrow A = 1$ $x = 1 \Rightarrow C = 1$ <p>coefft of x^2: $0 = -B + C \Rightarrow B = 1$</p>	M1 M1 B(2,1,0) [4]	clearing fractions substituting or equating coeffs for A,B or C $A = 1, B = 1, C = 1$ www
	<p>(iv)</p> $\int \frac{dx}{x^2(1-x)} dx = \int dt$ $\Rightarrow t = \int \left(\frac{1}{x^2} + \frac{1}{x} + \frac{1}{1-x} \right) dx$ $= -1/x + \ln x - \ln(1-x) + c$ <p>When $t = 0$, $x = 1/2 \Rightarrow 0 = -2 + \ln 1/2 - \ln 1/2 + c$</p> $\Rightarrow c = 2.$ $\Rightarrow t = -1/x + \ln x - \ln(1-x) + 2$ $= 2 + \ln \frac{x}{1-x} - \frac{1}{x} *$	M1 B1 B1 M1 E1 [5]	separating variables $-1/x + \dots$ $\ln x - \ln(1-x)$ ft their A,B,C substituting initial conditions
	<p>(v)</p> $t = 2 + \ln \frac{3/4}{1-3/4} - \frac{1}{3/4} = \ln 3 + \frac{2}{3} = 1.77 \text{ yrs}$	M1A1 [2]	